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Simulation of CANDU Reactor Transients Using Hierarchical Super Nodal Analysis

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ABSTRACT

In order to reduce the computing costs of the time-dependent diffusion calculations without introducing unacceptable inaccuracies in the solution, a hierarchical space-time kinetics procedure which is a combination of NET (Nodal Equivalence Theory) and QS (Quasi-Static method) is being developed. The time scales are divided into a three level hierarchy. On the smaller time step, the point kinetics equations are solved. At intermediate time steps, a coarse representation of the reactor core is solved using NET. Finally, full fine mesh calculations are performed over a large time scale. This scheme appears to have the potential of simulating demanding transients with lower computational cost than conventional methods. Detailed investigations are being carried out to reproduce the total power and flux distribution in coarse level calculations.

1. Introduction

The behavior of a nuclear reactor core during a transient is adequately described by space-time kinetics equations

$$[v]^{-1}\frac{\partial}{\partial t}[\Phi] = \vec{\nabla}\cdot[D]\vec{\nabla}[\Phi] - [\Sigma][\Phi] + (1-\beta)\left[\chi_p\right]\left[\vartheta\Sigma_f\right]^{\mathrm{T}}[\Phi] + \sum_{n=1}^{D}\left[\chi_{dn}\right]\lambda_n C_n \qquad (1)$$

$$\frac{\partial}{\partial t}C_n = \beta_n\left[\vartheta\Sigma_f\right]^{\mathrm{T}}[\Phi] - \lambda_n C_n \qquad (2)$$

where notations are standard. The solution of these equations can answer safety questions which arise in conjuction with actual and hypothetical accident scenarios. However, accurate three-dimensional solution of these equations is costly. Recently, in order to reduce the computing costs of the space-time calculations without introducing unacceptable inaccuracies in the solution, a hierarchical space-time kinetics procedure was developed[1] , [2] . The goal of the present work is to examine the computational merits of this procedure for realistic CANDU-6 transient simulations.

The following two sections provide an overall review of the method and its implementation in the computer code NDF[2] . In section 4, we present the numerical results obtained using this method for realistic CANDU-6 transients. The accuracy and speed of the method are then compared to those of classical improved quasi-static and direct fine mesh solutions obtained from NDF.

2. Description of the Method

The basis of the method is the space-time factorization which is the decomposition of the time-dependent neutron flux into the product of two functions: an amplitude function that depends only on time and a shape function that depends on space and energy as well as time:

$$[\Phi(\vec{r},t)] = [S(\vec{r},t)]\,T(t)\,, \tag{3}$$

where $S(\vec{r},t)$ is the shape function and $T(t)$ is the amplitude function. Normally, it is considered that the shape function is weakly time dependent, and as result it does not need to be computed at every time step. By applying this relationship to space-time kinetics equations, two sets of equations can be obtained[3] :

- the point kinetics equations for the amplitude function,
- and the space-time equations for the shape function.

When simulating transients which deal with large and fast changes of neutron distribution, the improved quasi-static method becomes computationally as time-consuming as the conventional direct methods. In order to deal with these situations, the quasi-static shape function can be calculated in different node size levels where each node size level corresponds to a time step hierarchy. In this procedure, it is supposed that the shape function in the coarser nodes varies faster than those of the finer nodes. Therefore, the shape function in the coarser node should be calculated more often. By using nodal equivalence theory[4] [5] , the preservation of the reactor eigenvalue, all group reaction rates and surface currents for each coarse mesh is guaranteed. Based on this approach and considering just one intermediate coarse level between the fine mesh calculation and the point kinetics, a three time step hierarchy can be obtained. On the smallest of these time hierarchies the point kinetics equations are solved. The point kinetics parameters $\rho(t)$, $\beta(t)$, $\lambda(t)$, and $\beta_n(t)$ are all time dependent and defined by weighted integrals of the flux and cross sections over the coarse regions. These parameters must be updated at the end of each point kinetics time steps to reflect all possible cross section variations in the core. Since the number of coarse regions is small, these calculations are much less time consuming compared to classical improved quasi-static method. On the intermediate time step the shape function is integrated over coarse nodes. Coarse nodes are homogenized regions of the reactor therefore, the precision of this time step depends on how well equivalence parameters can be approximated to correctly reflect reactivity device movements in the core. Finally, after many coarse shape calculations, the full core fine shape is solved. The necessary information to solve the fine mesh shape function comes from last available point kinetics calculation and coarse shape distribution. However, in the realistic calculations, it is not practical to perform too many fine mesh calculations for the sole purpose of updating the equivalence parameters. To deal with this problem, different approximations can be considered:

1) The first method is to establish an equivalence parameter database. All devices are initially set to a reference position. Each device is then moved and set to a new position independently. Based on the resulting fine mesh calculation, all equivalence parameters are then calculated and saved in the database. This database will hopefully permit fewer fine mesh calculations on long transients such as those involving Xenon. However, numerous tests show that when many devices are present

in the core, simple superposition or linear interpolation of the tabulated values of cross sections and discontinuity factors result in large errors. These errors are mostly due to device interference effects that are not considered in the database generation. Moreover, the use of the steady-state flux distribution to generate equivalence parameters is the origin of the device interference effects because it cannot reflect the dynamic flux distortions due to presence of many reactivity device in the core.

2) The second proposed solution is to define the correction terms which are the differences between the exact values of the equivalence parameters at the end of the fine mesh calculation and those which are tabulated. These correction terms would then be added to tabulated values of the equivalence parameters during the next fine time step. This method sounds promising, especially for benchmark problem solving.

3) The third method is to consider the rate of variation of the dynamic reactivity calculated in the point kinetics level as a criterion for switching between different time hierarchies. The practical difficulty of this method is the sensitivity of the dynamic reactivity to floating point calculations. For example, the dynamic reactivity of an unperturbed core transient can vary by a factor of 10 between two consecutive point kinetics calculations while the physical meaning for both cases represents a dynamic reactivity equal to zero.

3. Code Implementation

The computer code NDF[2] has been developed based on the aforementioned solution procedure. The NDF code is able to perform complicated static and dynamic calculations related to control and safety devices in a CANDU-6 reactor using the direct method, classical improved quasi-static method, and hierarchical space-time kinetics based on super nodal homogenization. The NDF consists of three major modules: fine mesh, coarse mesh and point kinetics modules. For the fine level module, a mesh-centered finite difference method is applied to produce the heterogeneous or exact solution of a model of a real CANDU-6 reactor. For coarse calculations, the generalized equivalence theory[5] is chosen because no iterative process is needed to calculate the discontinuity factors. According to generalized equivalence theory, discontinuity factors are defined as ratios of the heterogeneous fluxes and coarse mesh homogeneous fluxes over the coarse mesh interface. For example in the x-direction, time-dependent discontinuity factors on the interval $[x_l, x_{l+1}]$ are

$$f_{gx}^{-}(t) = \frac{\Phi_{gx}^{\text{het}}(x_l, t)}{\Phi_{gx}^{\text{hom}}(x_l, t)} \quad , \quad f_{gx}^{-}(t) = \frac{\Phi_{gx}^{\text{het}}(x_{l+1}, t)}{\Phi_{gx}^{\text{hom}}(x_{l+1}, t)} \tag{4}$$

In the NDF coarse module, a mesh-centered finite difference scheme is once more chosen and applied over coarse regions of the reactor allowing discontinuity of surface fluxes for all the nodes. For the point kinetics module the generalized Runge-Kutta (GRK4A) method of the Kaps-Rentrop family is used[6] . To minimize the error due to use of this model, point kinetics parameters are calculated using the corresponding static adjoint weight function. Finally, a full implementation of the θ-method on both the fine and coarse levels has been carried out to deal with the time integration of space-time kinetics equations. Numerous tests show that an implicit scheme $(\theta = 1)$ is

sufficiently accurate for the time integration procedures. Further details of NDF implementation can be found[2] .

4. Numerical Results

To examine the computational merits of this procedure, two simulation cases are presented. The first case is a well-known three-dimensional CANDU benchmark problem[7] and the second one is a demanding transient of a CANDU-6 reactor, with the presence of all reactivity devices such as liquid zone controllers and adjuster rods. All the transients are initiated by perturbing a steady-state situation. It is thus assumed that when these perturbations occur, the flux and precursors concentrations are at their steady-state values. These steady-state conditions are obtained from an initial static calculation. Obtaining the static eigenvalue equal to unity is unlikely, hence, the fission cross section values in both the diffusion and precursor equations are divided by the initial static eigenvalue.

4.1 Three-Dimensional CANDU benchmark

This benchmark problem is a three dimensional reactor kinetics problem in a heavy water reactor which is similar to a LOCA. The transient starts by a linear decrease of the total thermal cross section in regions 5, 6, 10, 11, 17, 18, 22 expressed by:

$$\frac{\partial \Sigma_t}{\partial t} = \left\{ \begin{array}{ll} -1 * 10^{-4}\,(\mathrm{cm}\cdot\mathrm{s})^{-1}, & t \leq 0.4 \text{ sec} \\ -8.88889 * 10^{-6}, & t > 0.4 \text{ sec} \end{array} \right\} \tag{5}$$

After 0.6 seconds an incremental thermal total cross section is added to regions 2, 4, 7, 9, 12, 14, 16, 18, 19, 21, 22, 23 to simulate asymmetric insertion of absorbers at a constant velocity of 520.0 cm/s in the y-direction.

The incremental value of the absorber is 6.150x10^{-4}(cm^{-1}). The transient lasts 2.5 seconds. The hatched areas are regions through which absorbers are inserted (Figure 1). For the sake of brievity the physical properties of the 24 regions are not tabulated here. The problem is modeled with two energy groups and six precursor groups. A non-uniform 18x18x10 grid for the x, y, and z-directions respectively was used to represent the fine regions. For the coarse representation of the reactor, various configurations were used: 3x3x2, 5x5x4 coarse nodes in x, y and z-directions. The steady-state results include the eigenvalue and the maximum error in the super node flux. Table 1 shows that this error is less than 0.001%. Figure 2 represents a plot of relative total power versus time for the time step $dt = 0.001$ for both reference[7] and direct solutions.

Evidently, reproducing this exact solution (direct solution) at a lower computational cost is the ultimate motivation behind of the present work. To assess the merits of the proposed method numerous

tests were performed. Some of the selected cases are described in Table 2.

Close examination of the results (Figures 3 and 4) demonstrate that coarser nodes (Case 1) can lead to substantial errors in the calculations. These errors are possibly due to three major reasons:

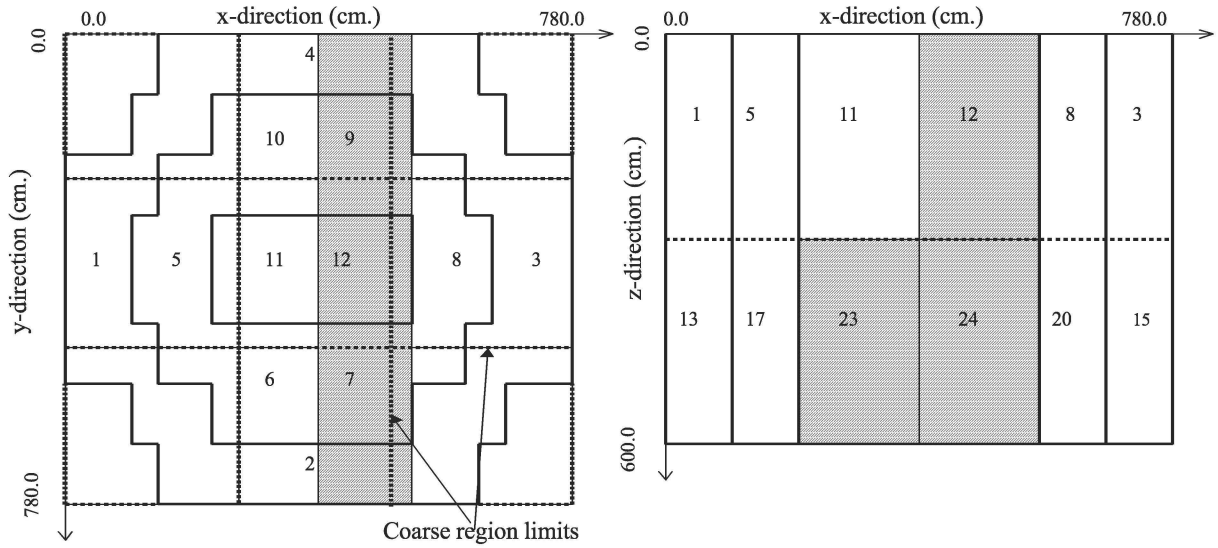

Figure 1: XY and XZ projections showing region assignments

Table 1: Results of Steady-State Calculations

Case	$\mathbf{k}_{eff}$	Max. flux error %
Reference[7]	1.00355	-
Fine (implicit)	1.00355	$\sim 10^{-4}$
3x3x2 Coarse node (implicit)	1.00355	$\sim 10^{-3}$
5x5x4 Coarse node (implicit)	1.00355	$\sim 10^{-4}$

Table 2: Description of the selected test cases

	Number of nodes	dt shape fine (sec)	N_{coarse}	N_{PK}	CPU time (sec)
Case 1	3 x 3 x 2	0.1	10	10	~300
Case 2	5 x 5 x 4	0.1	10	10	~380
Case 3	5 x 5 x 4	0.05	5	5	~ 400
IQS	18 x 18 x 10	0.01	–	10	~1200
Direct	18 x 18 x 10	0.001	–	–	~1500

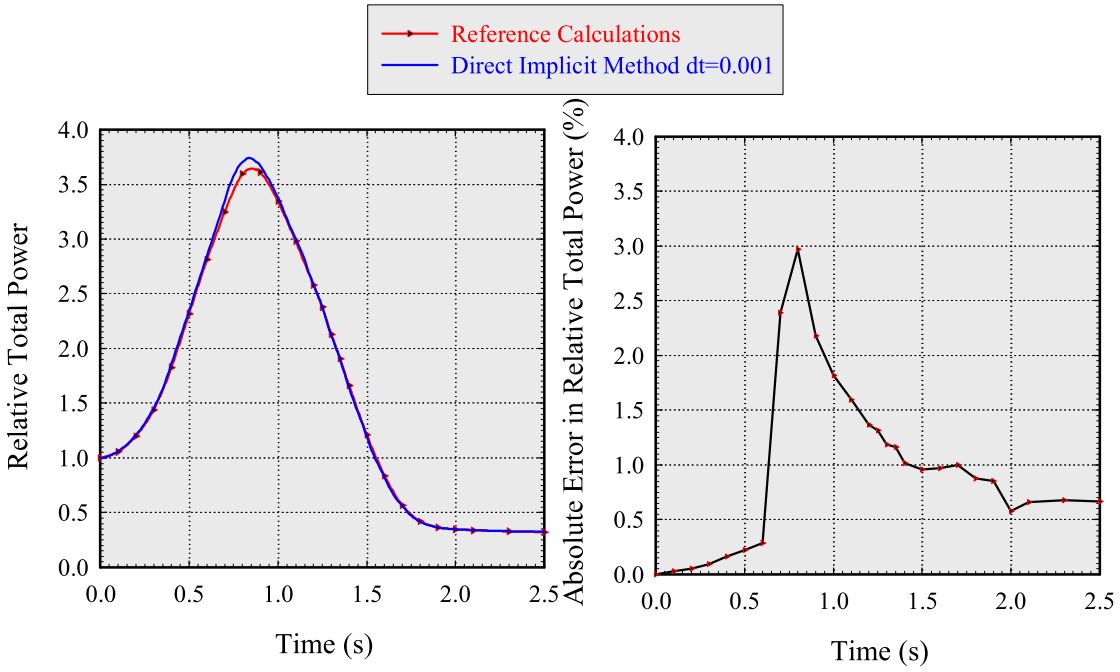

Figure 2: Relative total power vs. time for direct method and reference solution

Table 3: Summary of results for selected cases

	Method	dt shape fine (sec)	N_{coarse}	N_{PK}	CPU time (sec)
Case 1	Direct	0.01	–	–	~4500
Case 2	IQS	0.25	–	5	~4400
Case 3	HNK	1.00	5	4	~600
Case 4	HNK	0.75	5	3	~800
Case 5	HNK	0.6	3	4	~1000
Case 6	Direct	0.25	–	–	~1450

first, classical iterative error in flux calculations which are amplified in homogenized cross sections in the large volume of the reactor, second constant discontinuity factors which cannot correctly follow the fast and important distortions of flux after insertion of the absorber, and third the substantial error due to use of a simple dehomogenization technique for very coarse nodes. However, in practice, using smaller coarse nodes (Case 3), or a more complex dehomogenization technique (which is also more time-consuming), these errors could be successfully eliminated.

4.2 Three-Dimensional CANDU transient involving RRS

In the second example, the response of the CANDU-6 Reactor Regulating System (RRS) to a very substantial perturbation is simulated using the direct, the improved quasi-static and the hierarchical space-time kinetics methods. A CANDU-6 reactor is modeled with two energy groups and six precursor groups. A non-uniform 26 x 26 x 12 grid for the xy , and z-directions respectively was used to represent the fine regions in the direct, the improved quasi-static and hierarchical methods. It has been proven[2] that a 5 x 5 x 4 coarse grid in x, y and z-directions is adequate for the coarse representation of the reactor. In the initial steady-state conditions all the reactivity devices are set to their nominal positions. In the beginning of the reactor power transient two mechanical absorber (#1 and #2) rods are instantaneously inserted into the core. Table (3) and figures 5 and 6 show some of the results obtained for all three methods.

Further examination of the results (Figures 5 and 6) shows that the improved quasi-static method cannot be used efficiently for such transients (because of very fast local distortions). Moreover, it can be observed that the hierarchical nodal kinetics using constant discontinuity factors (Cases 3-5) all yield different results which depend on the time step dt used for the integration of the fine shape function. Case 5 which shows the results obtained using the new solution scheme developed as part of the current research is the only method that produces accurate results at an acceptable computational cost.

5. Conclusion

In an attempt to accurately predict the flux distribution and total power during a transient, a three-level space and time kinetics method based on nodal equivalence theory has been developed. This method can be considered as an extension to the improved quasi-static method. Extensive tests

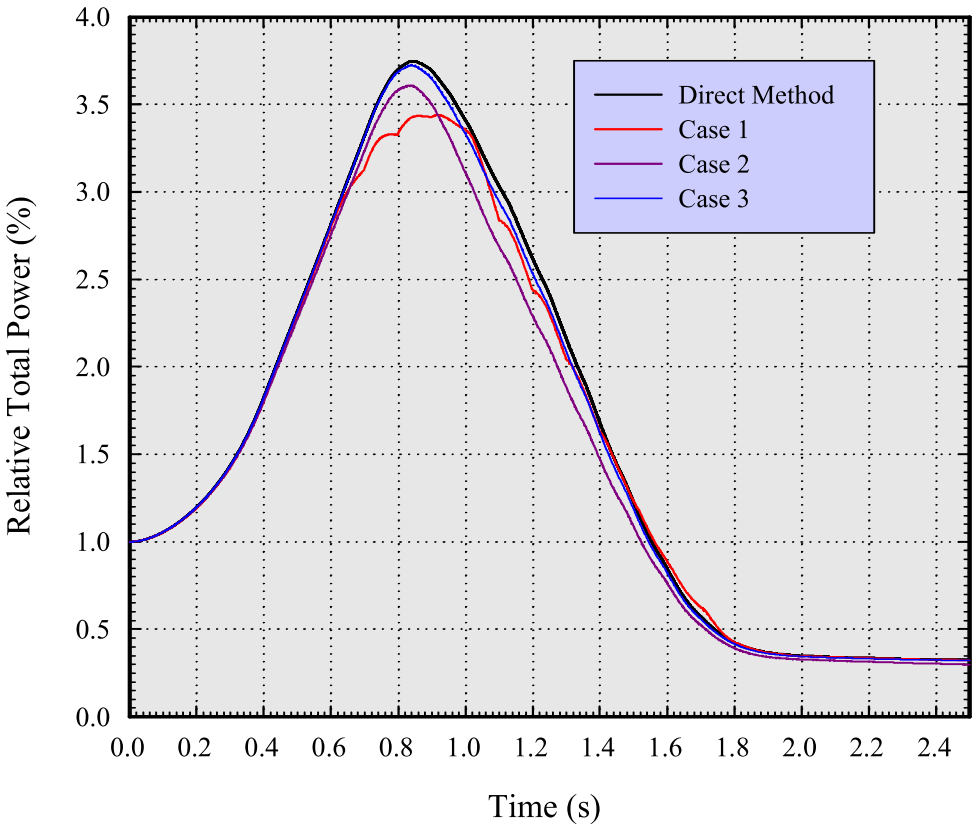

Figure 3: Relative total power vs. time for direct method and three-level quasi-static method

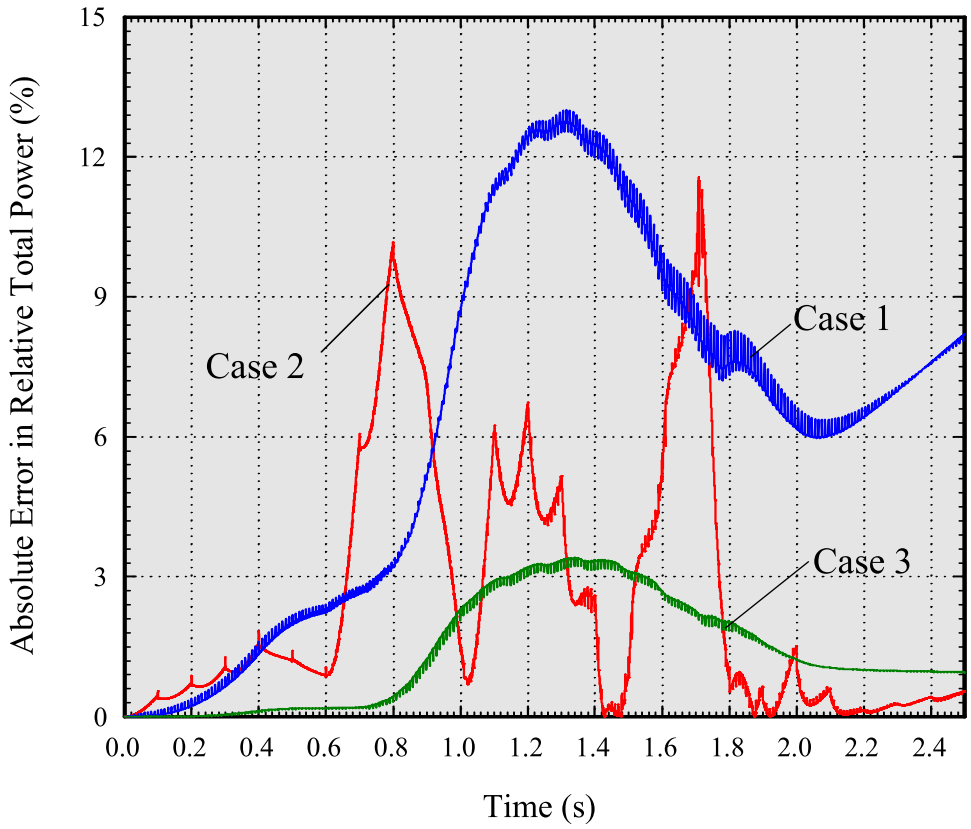

Figure 4: Absolute errors in relative power vs. time for cases 1, 2, 3

have shown that this method is much faster than the improved quasi-static method without loosing its efficiency. Using this method, the number of full core calculations carried out during a complicated transient can be substantially reduced. We expect that this method will substantially decrease the computational cost for the realistic CANDU-6 reactor simulations including full control and shutdown systems.

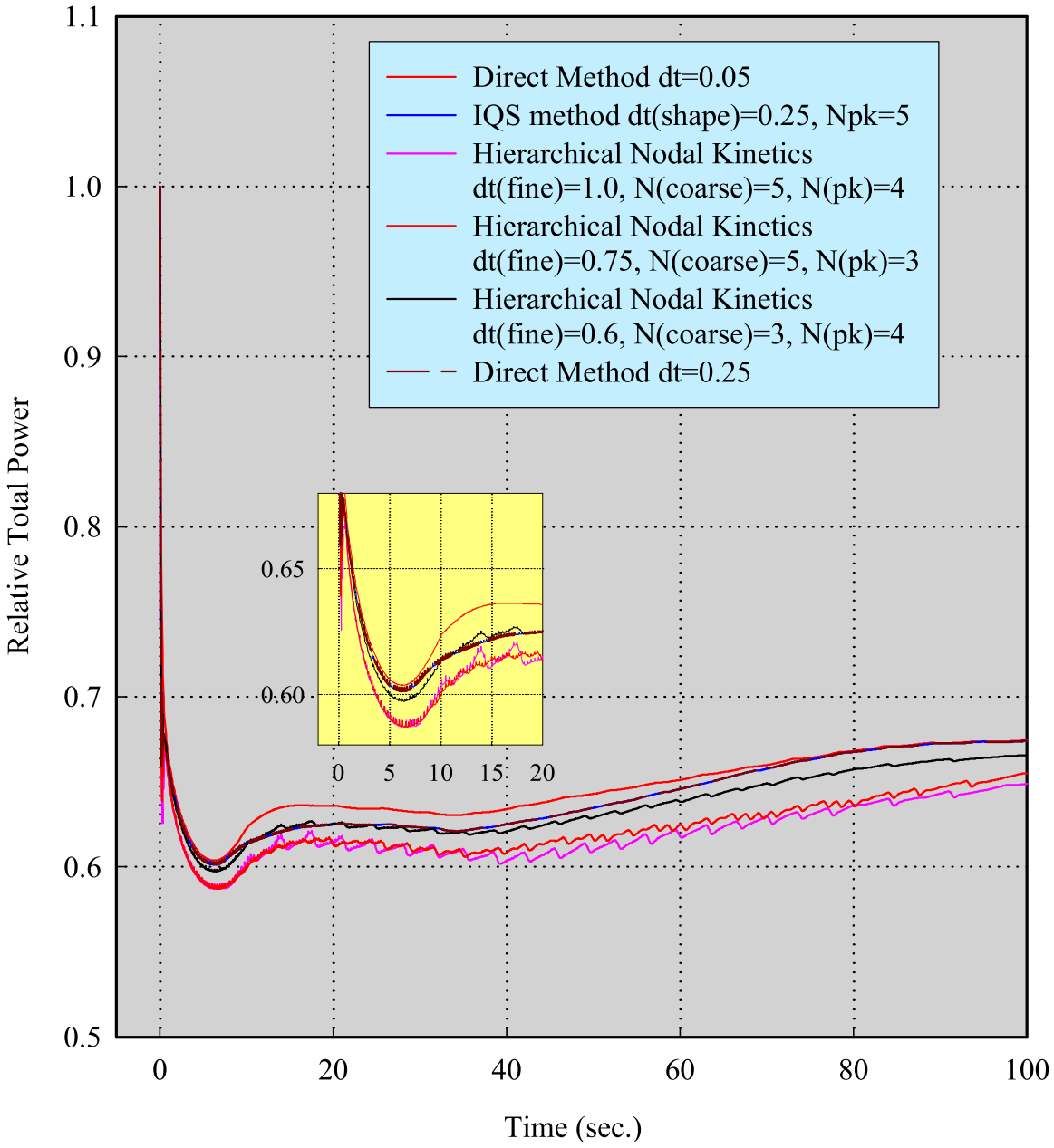

Figure 5: Relative total power for different cases

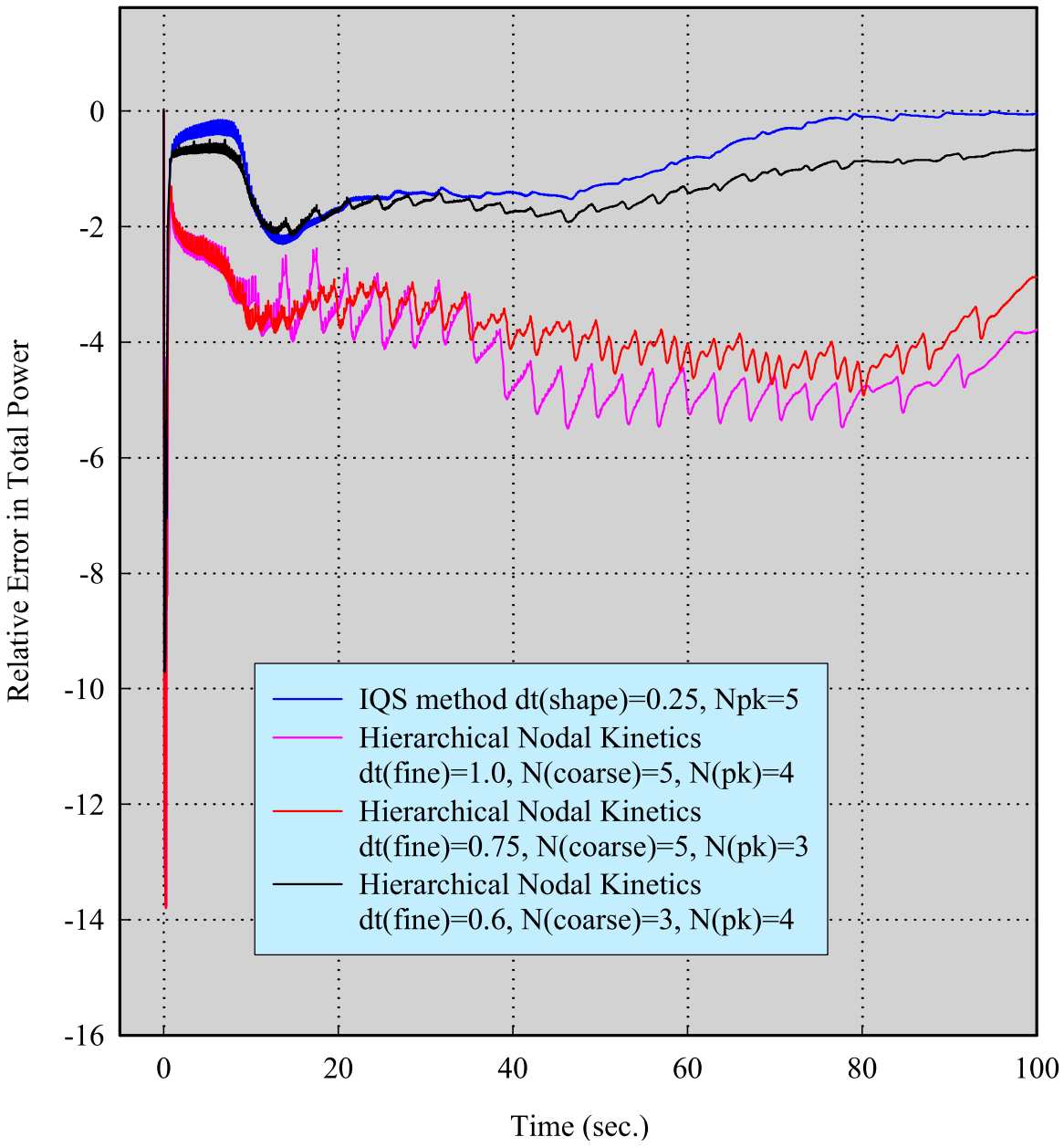

Figure 6: Error in relative total power for different cases

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