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# The Improved Quasi-Static Method vs the Direct Method: A Case Study for CANDU Reactor Transients

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## ABSTRACT

Among the large number of methods for the transient analysis of nuclear reactors, the improved quasi-static procedure is one of the most widely used. In recent years, substantial increase in both computer speed and memory has motivated a rethinking of the limitations of this method. The overall goal of the present work is a systematic comparison between the improved quasi-static and the direct method (mesh-centered finite difference) for realistic CANDU transient simulations. The emphasis is on the accuracy of the solutions as opposed to the computational speed. Using the computer code NDF, a typical realistic transient of CANDU reactor has been analyzed. In this transient the response of the reactor regulating system to a substantial local perturbation (sudden extraction of the five adjuster rods) has been simulated. It is shown that when updating the detector responses is of major importance, it is better to use a well-optimized direct method rather than the improved quasi-static method.

## 1. Introduction

Nuclear safety requires adequate knowledge of the behavior of reactor cores during transients. These transients cover a wide range of events from operational transients and fuel depletion to catastrophic accident excursions. The dynamic behavior of the neutrons during such transients is described by the time-dependent transport equations. However, the extension of numerical methods developed to solve transport equations to time-dependent problems which need thousands of calculations is very expensive from the point of the view of the computational resources required. Hence, time-dependent diffusion equations are normally considered as the starting point for transient calculations. Using standard notations, the time-dependent diffusion equations (also known as space-time kinetics equations) are

$$[v]^{-1}\frac{\partial}{\partial t}[\Phi] = \vec{\nabla}\cdot[D]\vec{\nabla}[\Phi] - [\Sigma][\Phi] + (1-\beta)[\chi_p][\nu\Sigma_f][\Phi] + \sum_{n=1}^{D}[\chi_{dn}]\lambda_n C_r \quad \text{(EQ 1)}$$

$$\frac{\partial C_n}{\partial t} = \beta_n[\nu\Sigma_f]^T[\Phi] - \lambda_n C_n \quad \text{(EQ 2)}$$

Substantial progress in the development of computational methods for treating both the spatial and temporal variables of these equations has been reported in the literature. The spatial methods can be categorized in three major groups: direct methods, space-time factorization methods, as well as modal and synthesis methods. In direct methods such as finite difference and nodal methods, space is directly discretized into elemental volumes. The spatially-discretized forms of the coupled diffusion equations are then obtained to produce a set of linear algebraic equations. Depending on the discretization methods which are applied different unknowns involving average flux, partial currents and net currents at the interfaces may be defined for each elemental volume. Space-time factorization methods such as point kinetics, quasi-static methods are based on the factorization of the space and time dependent flux into two parts. One part is only time-dependent and the other part is space- and weakly time-dependent. Finally, synthesis and modal methods are based on approximation of the flux by a linear combination of predetermined time-dependent spatial distributions. Time integration methods involve many methods such as the $\Theta$ -method, ADI (alternating direction implicit) method, stiffness confinement method, and the family of Runge-Kutta methods.

## 2. Improved Quasi-static Method

The basis of the method is the space-time factorization which is the decomposition of the time-dependent neutron flux into the product of two functions: an amplitude function that depends only on time and a shape function that depends on space and energy as well as time [1], [2]:

$$[\Phi(\vec{r},t)] = [S(\vec{r},t)]T(t) \quad \text{(EQ 3)}$$

where $[S(\vec{r},t)]$ is the shape function and $T(t)$ is the amplitude function. One assumption is that the transient is initiated from a steady state solution at time $t_0$,.
In order to force a unique solution, normalizing the factors in equation (3) must be applied. The standard approach is that $T(t) = T_0 = \text{const}$ for $t \le t_0$, and requiring that

$$\int[w(\vec{r})]^T[V]^{-1}[S(\vec{r},t)]d\vec{r} = \gamma = 1 \text{ for all } t > t_0 \quad \text{(EQ 4)}$$

Here $[w(\vec{r})]$ is an arbitrary weighting function that is usually selected to be the solution of the static adjoint diffusion equation corresponding to the initial state. By applying relationships (3) and (4) to time-dependent diffusion equations, two sets of equations can be obtained, first the point kinetics equations for the amplitude function expressed by:

$$\frac{d}{dt}T(t) = \frac{\rho - \bar{\beta}(t)}{\Lambda(t)}T + \sum_{d=1}^{D} \lambda_d C_d \qquad \text{(EQ 5)}$$

$$\frac{d}{dt}C_d = \frac{\beta_d}{\Lambda}T - \lambda_d C_d \qquad \text{(EQ 6)}$$

and second the space-time equations for the shape function as follows:

$$[v]^{-1}\frac{\partial}{\partial t}[S] + [v]^{-1}[S]\frac{1}{T}\frac{\partial}{\partial t}[T] = \vec{\nabla}\cdot[D]\vec{\nabla}[S] - [\Sigma][S] + (1-\beta)[\chi_p][\nu\Sigma_f]^T[S] + \frac{1}{T}\sum_{d=1}^{D}[\chi_{dn}]\lambda_n C_n \qquad \text{(EQ 7)}$$

Up to this point, the improved quasi-static method does not introduce any error in the solution of the equations (1) and (2). However, advantage may be gained if one assumes that the time-dependence of the shape function for relatively large time steps is weak enough that it may be considered constant. Hence, the behavior of the reactor during these large time steps can be adequately described by the point kinetics equations which are solved more often on a smaller time scale. This is normally an acceptable approximation for mild transients. However, in the cases where fast local shape distortions occur, the full core shape equations must be solved often during the transient. In solving for the amplitude function, the point kinetics parameters in equations (5) and (6) must be updated as often as necessary to account for changes in the materials properties, device movement, and shape function. A practical algorithm used in the original implementation of the improved quasi-static scheme is to consider that shape function varies linearly over the largest time interval. However, this consideration can lead to numerical difficulties in the neighborhood of prompt criticality. To eliminate such difficulties, an additional layer of iterations has been introduced to yield a converged value of shape function which satisfies both relationships (4) and (7). Evidently, this additional iteration procedure makes the improved quasi-static method more time consuming but also more precise.

## 3. Direct Method

It has been proven[3] that within the large variety of direct methods, the standard mesh-centered finite difference (also known as coarse mesh finite difference) method, which is the lowest order of all nodal methods, is sufficient for the study of CANDU reactors with one mesh per cell discretization. The higher order approximations in the nodal method would only produce a marginal improvement of the flux solution. In the mesh-centered finite difference method, the space is discretized by imposing a computational mesh and providing that material properties be considered as homogenized across each mesh box. The net surface currents then are approximated by assuming that the flux varies linearly between the node centerpoint and the midpoint of any surface of the

node. Eliminating the surface fluxes by enforcing continuity of the net current and flux across each interface leads to equations involving only the node-averaged fluxes. The most important property of the mesh-centered finite difference method is that it can be shown to converge to the exact solution of the space-time kinetics solutions as mesh spacing becomes increasingly small. Once the spatial approximation is completed, an initial value problem for a coupled system of ordinary differential equations in the following form can be obtained:

$$\frac{d}{dt}\begin{bmatrix}\overline{\Phi}\\ C\end{bmatrix} = \begin{bmatrix}[H]_{11} & [H]_{12}\\ [H]_{21} & [H]_{22}\end{bmatrix}\begin{bmatrix}\overline{\Phi}\\ C\end{bmatrix} \qquad \text{(EQ 8)}$$

which can be solved using many available numerical procedures, among them the well-established $\Theta$ finite difference scheme.

## 4. Numerical Simulation

To compare the numerical performance of the direct and the improved quasi-static methods, the response of the CANDU-6 Reactor Regulating System (RRS) to a very substantial perturbation is simulated using both methods available in the computer code NDF[4] . The CANDU-6 reactor is modeled with two energy groups and six precursor groups. A non-uniform 26 x 26 x 12 grid for the $x$ , $y$ and $z$ -directions respectively was used to represent fine regions in both the direct method and the improved quasi-static method. Moreover, to minimize the error due to the use of the point kinetics model, all parameters are calculated using the corresponding static adjoint weight function. In the initial steady-state conditions all the reactivity devices are set to the nominal positions. In the beginning of the reactor power transition, the bank#1 (including the center and the 4 corner adjuster rods) is instantaneously extracted from the core. All mechanical control absorbers are then considered disabled. Hence, the light water zone control system is the only available reactivity control mechanism (In CANDU-6 reactors, the light water zone control system is designed to perform two main functions: first, bulk control which is control of gross power output and second, spatial control which is control of the flux power shape). Table 1, and Figures 1, 2, 3, 4 and 5 show the results obtained for both methods.

To keep the improved quasi-static method as computationally less time consuming as possible no additional layer of iterations over $\gamma$ (relationship 4) has been applied, the shape function has rather been simply re-normalized after each shape calculation. To take this re-normalization into account, the amplitude is then re-adjusted to preserve the relationship (3).

Detailed examination of the results prove that the updating of cross sections, point kinetics parameters and detectors responses after each point kinetics time step (based on the new reconstructed flux) can make the improved quasi-static method as time consuming as a well-optimized direct method. Moreover, it can be observed that for the improved quasi-static method using

$dt_{shape}$ greater than 0.5 seconds, the additional layer of iterations over $\gamma$ is necessary (figure 3 and 4) which in turn makes the method computationally more expensive. The last conclusion is that the direct method with $dt = 0.25$ gives the best results considering both speed of calculation and precision of the result.

**TABLE 1. Summary of the CPU Times**

| Method | $dt_{shape}$ or $dt_{direct}$ | Number of PK | Approximate CPU time (sec) |
|---|---|---|---|
| DIRECT | 0.01 | - | 16000 |
| DIRECT | 0.05 | - | 3400 |
| DIRECT | 0.1 | - | 1900 |
| DIRECT | 0.25 | - | 940 |
| DIRECT | 0.50 | - | 560 |
| IQS | 0.1 | 10 | 15600 |
| IQS | 0.25 | 5 | 4300 |
| IQS | 0.50 | 10 | 3300 |
| IQS | 1.0 | 20 | 3200 |

## 5. Conclusion

Updating the detector responses in a realistic CANDU-6 transient calculation which involves all reactivity devices and the reactor regulating system is of major importance. This updating must reflect the all cross section variations due to the moving devices. Hence, to obtain acceptable results from improved quasi-static method, the point kinetics parameters must be updated as often as possible. However, the computational costs for these updates will soon become significant. In general, it can be concluded that a well-optimized direct method is superior to the improved quasi-static method in both the speed of calculations and precision of the results.

## 6. References

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[2] H.L. Dodds,Jr., *Accuracy of the Quasistatic Method for Two-Dimensional Thermal Reactor Transients with Feedback*, Nucl. Sci. Eng., 59, 271-281, (1976).
[3] Koclas J., *Comparison of the Different Approximations Leading to Mesh Cenetered Finite Differences Starting from the Analytic Nodal Method*, Ann. Nucl. Energy, Vol. 25, No. 11, pp. 821-838, 1998.
[4] Kaveh, S., Koclas, J., Roy, R. *Three-level Space-Time Kinetics based on Super Nodal Analysis*, Proceedings, 19th Annual Conference, Vol. 2, Canadian Nuclear Society, (1998).

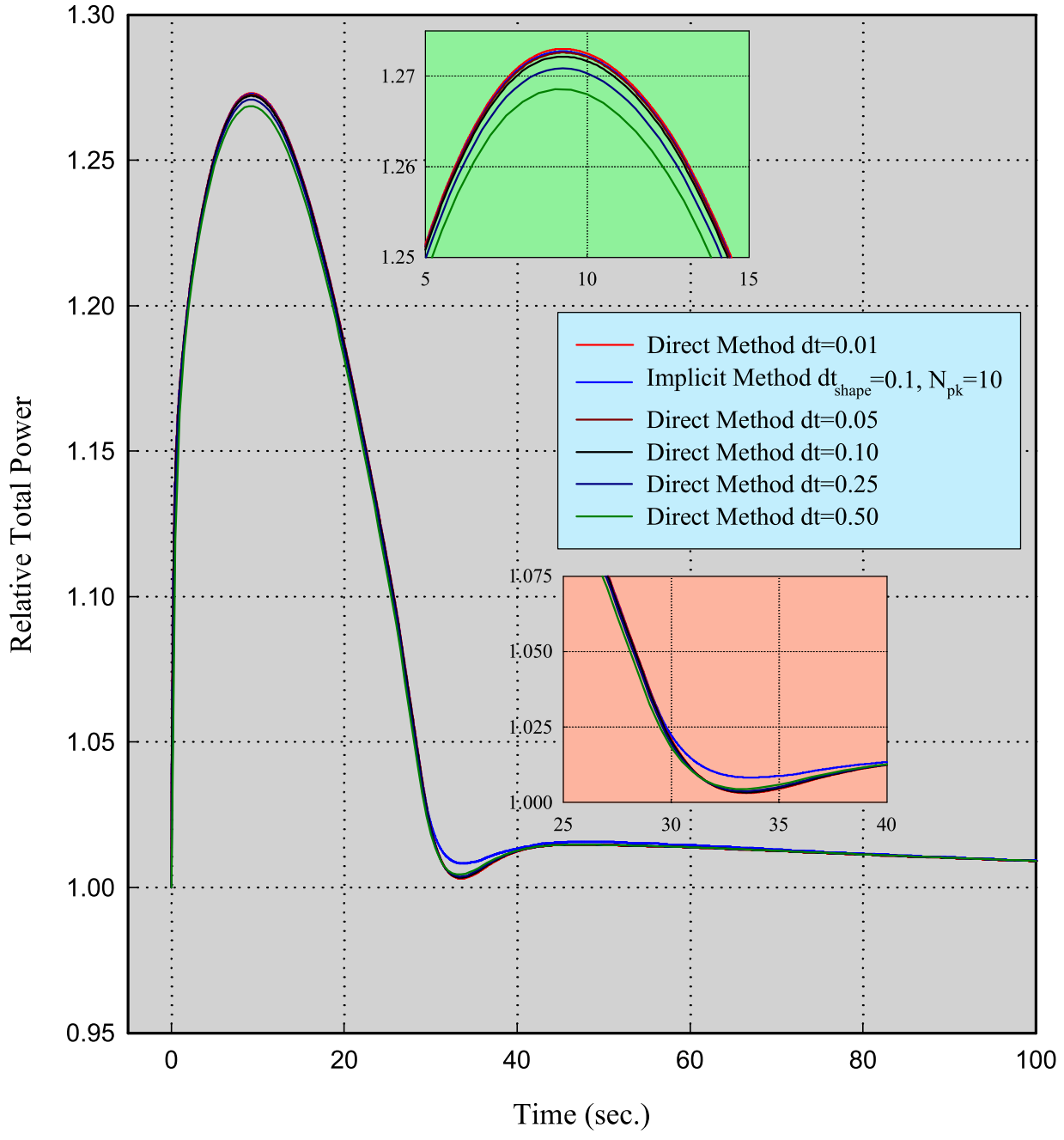


Figure 1: Relative total power for the direct method using different time steps

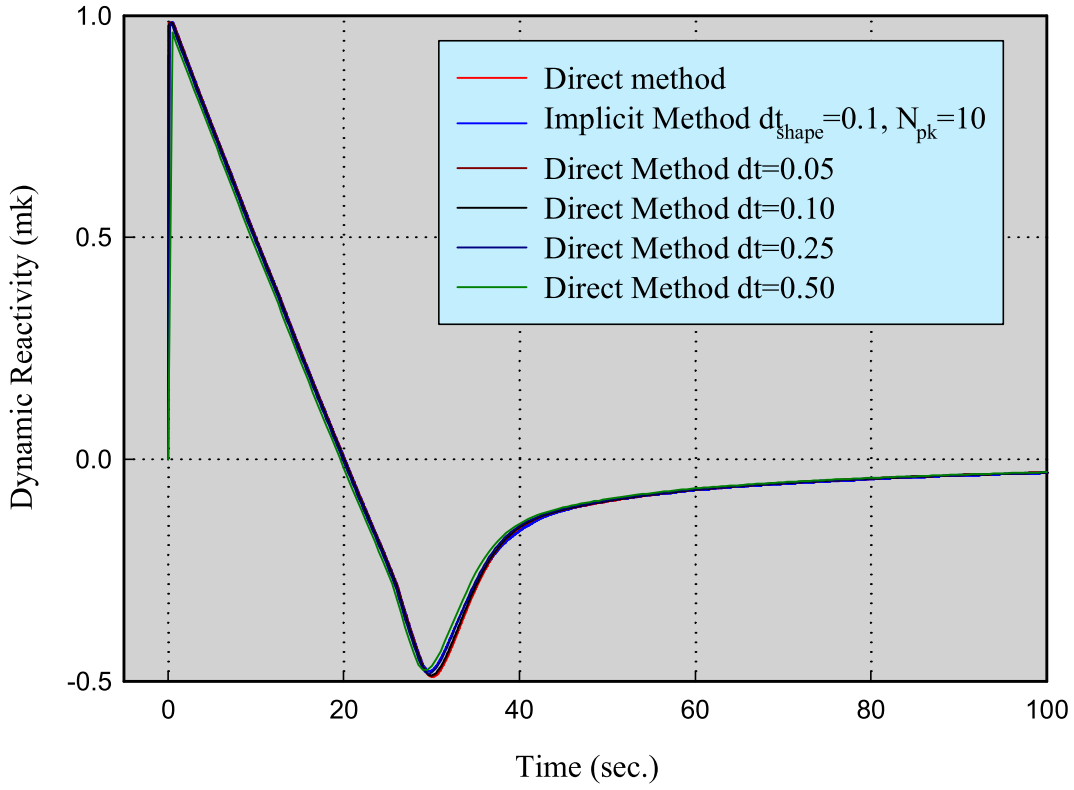


Figure 2: Dynamic reactivity for the direct method using different time steps

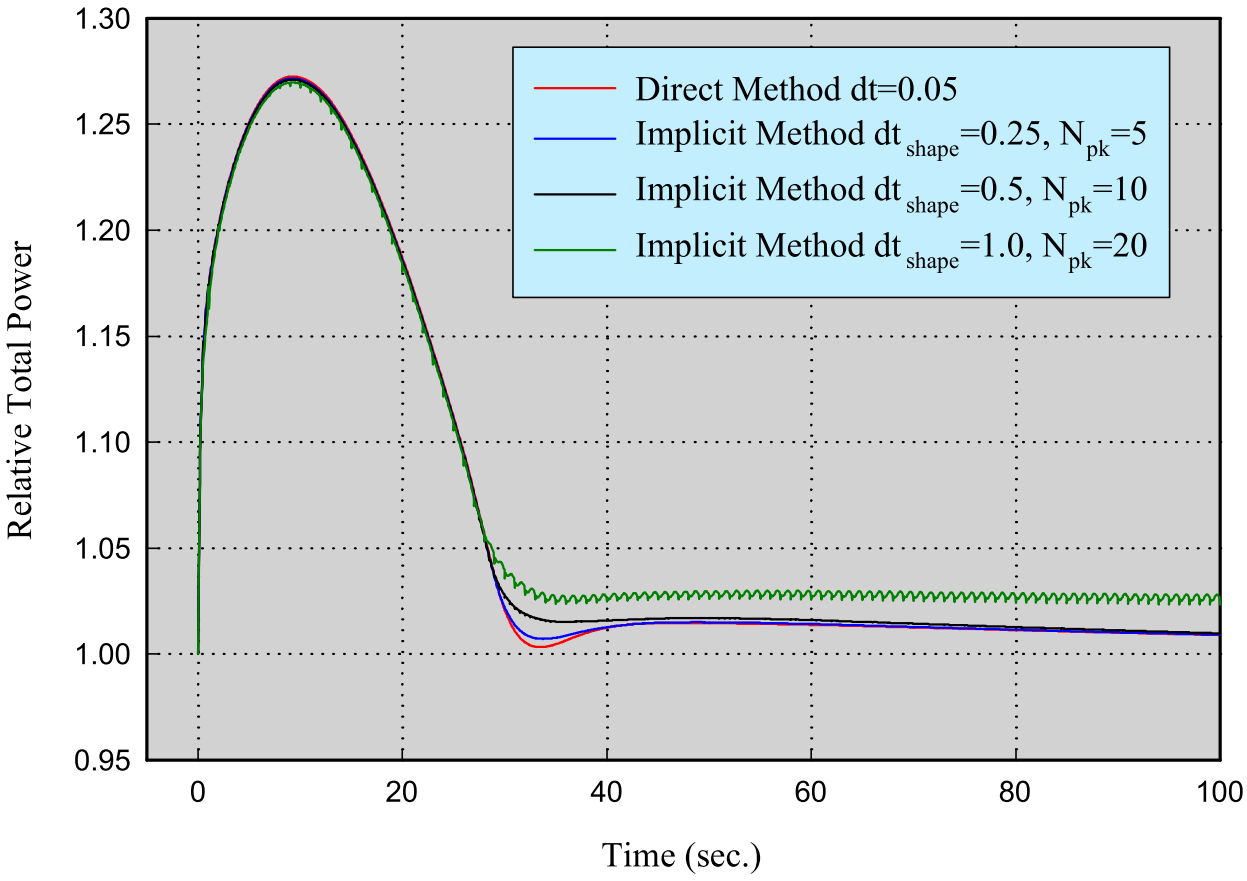


Figure 3: Relative total power for the improved quasi-static method using different time steps

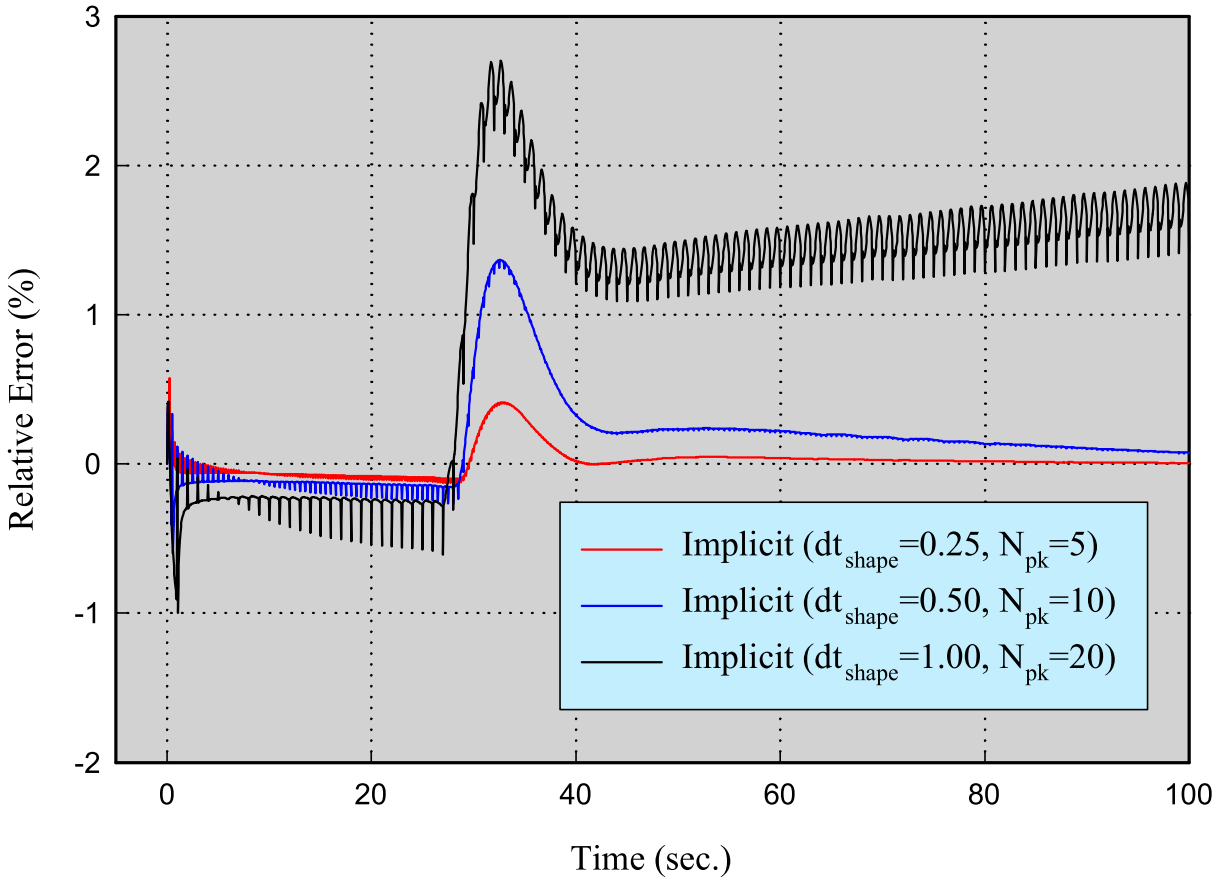


Figure 4: Relative error for the improved quasi-static method using different time steps

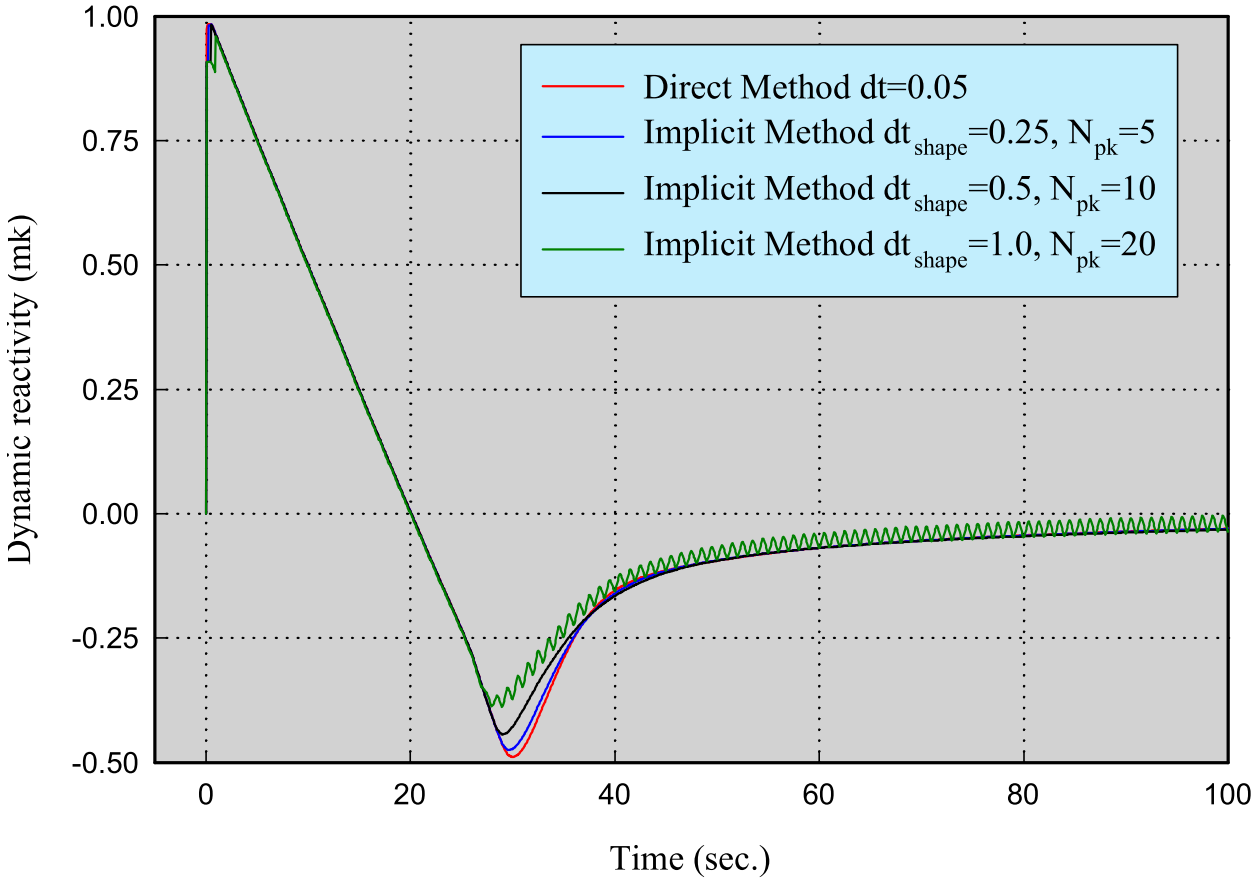


Figure 5: Dynamic reactivity for the improved quasi-static method using different time steps