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The Unavailability and Delay Time of an Action due to the Drift of the Instruments

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1. Introduction

In CANDU nuclear plants as well as other nuclear plants, important process parameters are continuously measured by instruments and compared to their setpoints for actions, such as control, protection and annunciation. Due to various reasons, however, the measured value may not be exactly the same as the real value. This discrepancy is called drift; the drifting of the measured value from the real value. Drift within a certain range is anticipated. Instrumental failure is classified as the drift mode with an unacceptable magnitude.

To a particular instrument, its drift is a random constant, which may be process dependent. To different instruments of the same type, their drifting constants are different and these constants show a probability distribution. Because each drifting constant may be process dependent, the drift distribution may also be process dependent.

The existence of the probabilistically distributed drifts in the instruments causes the measured value being different from the real value, resulting in the unavailability of the instruments and therefore, the actions which the instruments are initiating.

The prediction of unavailability of every protection action is required for the safe operation of a nuclear plant. The unavailability of certain actions, ECC injection for example, are required to be controlled below a very smaller number. The calculation of the future unavailability is normally carried out by using the classic event tree method. In the classical event tree method, the unavailability (the probability of the top event) is obtained from that of a series of intermediate events which in turn, are determined by the probabilities of the basic events. The probabilities of all events are treated as constant.

Because the drift distribution may be process dependent, the unavailability obtained by omitting this dependency may not be able to reflect the reality. Moreover, the instrumental drift affecting the initiation of an action is a function of process parameter. Neglecting this functional relationship tends to yield erroneous prediction.

To improve the accuracy of the unavailability prediction, the relationship between the instrumental drift and the process parameters needs to be taking into account. This is the motivation of the present work. In the present work, the unavailability of an instrument and an action are computed dynamically. First, the unavailability of an instrument due to its drift is developed. The probability distribution of the drift is then determined from the test of the instrument. The unavailability of an action initiated by the instrument or instrument loop based on the principle of single or majority vote is then derived. Finally, the unavailability of the action due to instrumental drift is calculated, and delay time for the initiation of the action is determined. The developed formula is applied to different instruments and instrument loops. The computational results are compared with that from the classic event tree method and other available data.

2. Unavailability of Instruments and Drift Distribution

Let the measured and the real value of a process parameter be denoted by $X_m(t)$ and $X_r(t)$, respectively, and the instrumental drift, by x_d. Then $X_m(t)=X_r(t)+x_d$.The unavailability of an instrument is the probability of its drift being beyond certain range, say $\pm\lambda$.

$$p_I(|x_d|=|X_m(t)-X_r(t)|>\lambda)=p_I(x_d\leq-\lambda)+p_I(x_d\geq\lambda)=\int_{-\infty}^{-\lambda}f_d(x_d,t)dx_d+\int_{\lambda}^{\infty}f_d(x_d,t)dx_d \tag{1}$$

where $f_d(x_d,t)$ is the probability density function of drift, and the subscript *I* stands for instrument. The first term is the unavailability due to low drifts, and the second term, high drifts.

An action is initiated if the measured value of the process parameter is larger (or smaller) than its setpoint, denoted by X_s. Since $X_m=X_r+x_d$, the difference between measured value and the setpoint is: $X_m-X_s= X_r(t)+x_d-X_s= x_d+(X_r(t)-X_s)= x_d+\lambda(t)$, where $\lambda(t)=X_r(t)-X_s$.

In the case of actions being initiated when $X_m\geq X_s$ (called first kind action hereafter), the drifts affect the initiation of the actions are low drifts $x_d\leq-\lambda= -(X_r(t)-X_s)$. Based on Eq. (1), the unavailability of the instrument, denoted by $p_{1,I}$, is:

$$p_{1,I}=p_I(x_d\leq-\lambda(t))=\int_{-\infty}^{-\lambda(t)}f_d(x_d,t)dx_d$$

In the case of actions being initiated when $X_m\leq X_s$ (called the second kind action hereafter), the drifts affect the initiation of the action are high drifts $x_d\geq-\lambda= -(X_r(t)-X_s)$, and the unavailability of the instrument is:

$$p_{2,I}=p_I(x_d\geq-\lambda(t))=\int_{-\lambda(t)}^{\infty}f_d(x_d,t)dx_d$$

The unavailability can be determined once the drift distribution function is found.

The distribution function $p_I(x_d,t)$ depends on the type of the instrument. Raw data from the plant tests can be used to determine it. Normally, each instrument is calibrated before it is put in service, and tested after it has been in service for a certain period of time. Various criteria, set according to the industry standard, are used to classify the drift modes before the tests. During the tests, the difference between a measured value and its source value may be observed, and the number of occurrence of drift of all modes is recorded. Average observed rate of each drift mode per thousand component in-service years is then calculated. Not losing generality, let's assume that the criteria for drift modes are denoted by $\pm c_0$, $\pm c_1$, $\pm c_i$..$\pm c_m$ with $0<c_0<c_1<..<c_i<..<c_m<\infty$ (when absolute drift>c_m, the instrument is declared out of function). Following information about the

$$p_I(-c_0 < x_d < c_0, t) = k_{I,0}(t), \qquad p_I(c_{i-1} < x_d < c_i, t) = k_{I,i}(t),$$

distribution function can be obtained from the test data:

$$p_I(-c_i < x_d < -c_{i-1}, t) = k_{I,-i}(t),$$

where the k's are the average observed rates per thousand in-service years. The subscript "i", indicates a drift mode and the superscript "-" and "+", drift low and high, respectively. All k's are less than 1, and they are calculated by: k=1000(n/N),where n is the total number of observing drift of the specified drift mode and N is the total cumulative component in-service years. The probability of the instrument out of function is:

$$p_I(x_d < -c_m, t) = k_{I,-\infty} \qquad p_I(c_m < x_d, t) = k_{I,\infty}$$

At time t=0, all instruments are calibrated and all drifts, except those within the limit of the calibration accuracy, are zero, so $k_{I,-i}(0)=k_{I,i}(0)=0$ (i=1,..M,∞) and $k_{I,0}(0)=1$. The k's are constrained by: $(k_{I,-\infty}+k_{I,-m}+\ldots k_{I,-2}+k_{I,-1}+k_{I,0}+k_{I,1}+k_{I,2}+\ldots+k_{I,m}+k_{I,\infty}) = 1$ at any time.

Normally c_0, c_1, c_i..c_m are very small, thus it is reasonable to assume that the distribution function can be represented by piece-wise linear functions, hence the probability of low and high drift in an instrument I is:

$$p_I(x_d < -\lambda(t)) = \begin{cases} \sum_{j=i+1}^{\infty} k_{I,-j}(t) + \dfrac{k_{I,-i}(t)}{c_i - c_{i-1}}(c_i - \lambda(t)) & -c_i < -\lambda(t) < -c_{i-1} \\ \sum_{j=1}^{\infty} k_{I,-j}(t) + \dfrac{k_{I,0}(t)}{c_0 + c_0}(c_0 - \lambda(t)) & -c_0 < -\lambda(t) < c_0 \\ \sum_{j=1}^{\infty} k_{I,-j}(t) + k_0(t) + \sum_{j=1}^{i-1} k_{I,j}(t) + \dfrac{k_{I,i}(t)}{c_i - c_{i-1}}(-c_{i-1} - \lambda(t)) & c_{i-1} < -\lambda(t) < c_i \end{cases}$$

and

$$p_I(x_d > -\lambda(t)) = \begin{cases} \sum_{j=1}^{\infty} k_{I,j}(t) + k_{I,0} + \sum_{j=1}^{i-1} k_{I,-j}(t) + \dfrac{k_{I,-i}(t)}{c_i - c_{i-1}}(\lambda(t) - c_{i-1}) & -c_i < -\lambda(t) < -c_{i-1} \\ \sum_{j=1}^{\infty} k_{I,j}(t) + \dfrac{k_{I,0}(t)}{c_0 + c_0}(\lambda(t) + c_0) & -c_0 < -\lambda(t) < c_0 \\ \sum_{j=i+1}^{\infty} k_{I,j}(t) + \dfrac{k_{I,i}(t)}{c_i - c_{i-1}}(\lambda(t) + c_i) & c_{i-1} < -\lambda(t) < c_i \end{cases}$$

Since the parameter $\lambda(t)$ varies with the real value $X_r(t)$, the unavailability of the instrument and its initiating action are functions of $X_r(t)$.

Based on the probability of the low and high drift, the unavailability as well as the delay time of an instrument and an action can be computed.

3. Unavailability of Actions

An instrument loop or loops is normally used to measure the process parameter and to initiate an action. An instrument loop consists of two instruments, a transmitter and a current alarm element. The transmitter (represented by subscript T, T=PT for pressure transmitter, T=DPT for differential pressure transmitter and T=TT for temperature transmitter) measures the parameter and the current alarm unit (represented by subscript A) activates the contacts. The transmitter and the current alarm unit are connected in a series. By considering that the unavailability of both instruments are independent, the unavailability of the first kind action initiated by single vote, denoted by $p_{1,TA}$ can be obtained as:

$$p_{1,TA} = p_{1,T}(x_d \leq -\lambda(t)) + p_{1,A}(x_d \leq -\lambda(t)) - p_{1,T}(x_d \leq -\lambda(t))p_{1,A}(x_d \leq -\lambda(t)) \qquad (2)$$

and the unavailability of the second kind action is denoted by $p_{2,TA}$ and obtained as:

$$p_{2,TA} = p_{2,T}(x_d \ ? \ \lambda(t)) + p_{2,A}(x_d \ ? \ \lambda(t)) - p_{2,T}(x_d \ ? \ \lambda(t))p_{2,A}(x_d \ ? \ \lambda(t)) \qquad (3)$$

Many actions are initiated by a majority vote (normally, 2 out of 3) among three instrument loops. The unavailability of these actions can be determined by using the corresponding unavailability of the single vote action as:

(1) action initiated when $X_m \geq X_s$,

$$p_{1,TA,2/3} = 3p_{1,TA}^2 - 2p_{1,TA}^3$$

(2) action initiated when $X_m \leq X_s$,

$$p_{2,TA,2/3} = 3p_{2,TA}^2 - 2p_{2,TA}^3$$

4. The Delay of the Initiation of an Action

To the unavailability $p_{1,I}$, as the process parameter increases towards the setpoint and exceeds it, $-\lambda$ changes from positive to negative. As $-\lambda$ becomes smaller and smaller, $p_{1,I}$ decreases and finally reduces to $k_{I,-\infty}$. To the unavailability $p_{2,I}$, as the parameter decreases towards the setpoint and falls below it, $-\lambda$ changes from negative to positive. As $-\lambda$

becomes larger and larger, $p_{2,I}$ decreases and finally reduces to $k_{I,\infty}$. If the instruments are not completely failed ($k_{I,-\infty}\neq 1$ and $k_{I,\infty}\neq 1$), the action is initiated as a delay time has elapsed. The larger the drift, the longer the delay time. Let TD be the delay time of an action. The time t starts at λ=0. The probability of TD>t is the probability that the drifts are still affecting the initiation of the action, so it is equal to the unavailability of the action.

The delay time of the first and the second kind action due to drifts in an instrument drift is therefore obtained, respectively, as [1]:

$$TD_{1,I} = \int_0^{\infty} t\frac{dp_{1,I}}{dt}dt = -\int_0^{\infty} p_{1,I}dt = -\int_0^{-\infty} p_{1,I}(\frac{dX_r}{dt})^{-1}d\lambda = -(\frac{dX_r}{dt})_m^{-1}\int_0^{-\infty} p_{1,I}d\lambda$$
$$= -(\frac{dX_r}{dt})_m^{-1}[c_0(\sum_{j=0}^{m} k_{I,-j} - \frac{3k_{I,0}}{4}) + \sum_{i=1}^{m}(c_i - c_{i-1})(\sum_{j=i}^{m} k_{I,-j} - \frac{k_{I,-i}}{2})] \quad (4)$$

$$TD_{2,I} = \int_0^{\infty} t\frac{dp_{2,I}}{dt}dt = -\int_0^{\infty} p_{2,I}dt = -\int_0^{\infty} p_{2,I}(\frac{dX_r}{dt})^{-1}d\lambda = -(\frac{dX_r}{dt})_m^{-1}\int_0^{\infty} p_{2,I}d\lambda$$
$$= (\frac{dX_r}{dt})_m^{-1}[c_0(\sum_{j=0}^{m} k_{I,j} - \frac{3k_{I,0}}{4}) + \sum_{i=1}^{m}(c_i - c_{i-1})(\sum_{j=i}^{m} k_{I,j} - \frac{k_{I,i}}{2})] \quad (5)$$

where $(dX_r/dt)_m$ is the mean of the rate of change of X_r, and negative means delay.

The response time of an instrument due to its drift alone can be obtained by setting $(dX_r/dt)_m=1$ in Eq. (4) and (5).

It can be seen from the above expressions that all TD's decrease as the rate of change of $X_r(t)$ increases. So, the fast the change of the process parameter, the shorter the delay time of the action.

By following the same procedure, the delay time of an action initiated by single or majority vote of instrument loop(s) can be determined, however, the calculation is tedious and resulting expressions are length. Considering that all unavailability are between zero and one, following relations can be established to estimate the delay time of the actions initiated by single vote:

$$TD_{1,TA} = \int_0^{\infty} p_{1,TA}dt = \int_0^{\infty}[p_{1,T}(x_d \le -\lambda(t)) + p_{1,A}(x_d \le -\lambda(t)) - p_{1,T}(x_d \le -\lambda(t))p_{1,A}(x_d \le -\lambda(t))]dt$$
$$\le TD_{1,T} + TD_{1,A} - \min[TD_{1,T}, TD_{1,A}] = \max[TD_{1,T}, TD_{1,A}]$$
$$TD_{1,TA} = \int_0^{\infty} p_{1,TA}dt = \int_0^{\infty}[p_{1,T}(x_d \le -\lambda(t)) + p_{1,A}(x_d \le -\lambda(t)) - p_{1,T}(x_d \le -\lambda(t))p_{1,A}(x_d \le -\lambda(t))]dt$$
$$?\ TD_{1,T} + TD_{1,A} - \max[TD_{1,T}, TD_{1,A}] = \min[TD_{1,T}, TD_{1,A}]$$

$$\min[TD_{2,T}, TD_{2,A}] \le TD_{2,TA} \le \max[TD_{2,T}, TD_{2,A}]$$

These inequalities can be used to estimate the delay time of the actions initiated by majority vote.

$$TD_{1,TA,2/3} = \int_0^{\infty}[3p_{1,TA}^2 - 2p_{1,TA}^3]dt \le 3\int_0^{\infty} p_{1,TA}^2 dt \le 3\int_0^{\infty} p_{1,TA} = 3TD_{1,TA}$$

$$TD_{1,TA,2/3} = \int_0^{\times} [3p_{1,TA}^2 - 2p_{1,TA}^3]dt \; ? \; \int_0^{\times} [3p_{1,TA}^2 - 2p_{1,TA}^2]dt = \int_0^{\times} p_{1,TA}^2 dt \; ? \; [\int_0^{\times} p_{1,TA} dt]^2 = TD_{1,TA}^2$$

$$TD_{2,TA}^2 \leq TD_{2,TA,2/3} \leq 3TD_{2,TA}$$

5. Application

The unavailability and delay time derived in section 3 and 4 are applied in this section to instrument loop consisting of a transmitter (pressure, differential pressure, temperature) and current alarm unit.

A sample of the criteria of different drift modes for different instruments is given in TABLE-1.

TABLE-1. The criteria for drift modes of different instrument [2]

Instrument	(c_0)	(c_1)	($c_2 = c_M$)
Pressure transmitters (PT)	0.2% Upper range limit + 0.25% Span	5% Span	Erratic or full span
Differential pressure transmitters (DPT)	0.2% Upper range limit + 0.25% Span	5% Span	Erratic or full span
Temperature transmitters (TT)	0.2% Upper range limit+0.5% Span	0.5% Span	Erratic or full span
Current alarm unit (A)	1% FP setpoint	5% FP setpoint	Full span

By using these criteria, the drift rate of each drift mode can be obtained from the tests. TABLE-2 shows a probability distribution of the drifts in some transmitters.

TABLE-2 The probability of drifts of different instruments [2]

Instrument	Fail low ($-\infty, -c_2$)	Large drift low($-c_2, -c_1$)	Small drift low($-c_1, c_0$)	Small drift high (c_0, c_1)	Large drift high(c_1, c_2)	Fail high (c_2, ∞)
PT	2.595E-3	3.46E-3	11.25E-3	6.93E-3	18.18E-3	2.595E-3
DPT	3.29E-3	16.04E-3	25.53E-3	12.40E-3	13.86E-3	3.29E-3
TT	9.4E-4	5.64E-3	15.05E-3	15.05E-3	5.65E-3	9.4E-4
A	4.84E-3	6.98E-3	2.69E-3	0.0	4.3E-3	8.06E-3

As an example, the unavailability $p_{1,TA}$, $p_{2,TA}$, $p_{1,TA,2/3}$, $p_{2,TA,2/3}$ are computed for T=PT, DPT and TT by using this probability distribution. The results are presented in Figure 1 to 6, respectively.

The classic event tree method does not consider the dependency of the unavailability on the process parameter, thus the unavailability is always a constant. For an action signaled by a PT-A loop, initiated by a single vote, its unavailability is calculated as:

$$p_{1,PTA} = p_{1,PT} + p_{1,A} - p_{1,PT}p_{1,A}$$
$$= (0.002595 + 0.00346 + 0.01125) + (0.00484 + 0.00698 + 0.00269) - (0.002595 + 0.00346 + 0.01125) \leftrightarrow$$
$$(0.00484 + 0.00698 + 0.00269) = 0.031564$$
$$p_{2,PTA} = p_{2,PT} + p_{2,A} - p_{2,PT}p_{2,A}$$
$$= (0.002595 + 0.00693 + 0.01818) + (0.0 + 0.0043 + 0.00806) - (0.002595 + 0.00693 + 0.01818) \leftrightarrow$$
$$(0.0 + 0.0043 + 0.00806) = 0.03972$$

For an action signaled by three PT-A loops and initiated by majority votes, the unavailability of the action is obtained as:

$$p_{1,PTA,2/3} = 3p_{1,PTA}^2 - 2p_{1,PTA}^3 = 3(0.03152)^2 - 2(0.03152)^3 = 0.002918$$
$$p_{2,PTA,2/3} = 3p_{2,PTA}^2 - 2p_{21,PTA}^3 = 3(0.03972)^2 - 2(0.03972)^3 = 0.004608$$

Similarly, the unavailability of the actions initiated by DPT-A loop(s) or TT-A loop(s) can be calculated. The results are listed in Table 3.

Table 3 The unavailability from classic event tree method

	PT-A loop	DPT-A loop	TT-A loop
First kind action (single vote)	0.031564	0.05872	0.03645
First kind action (majority vote)	0.002918	0.009939	0.003889
Second kind action (single vote)	0.039723	0.04154	0.03372
Second kind action (majority vote)	0.004608	0.005033	0.003334

The unavailability form the classic event tree method are compared with that of the present work in Figure 1 to 6. It can be seen that the present unavailability decreases as X_r deviates from its setpoint X_s. This is due to the fact that more and more drifts are compensated by the process parameter. The classic event tree method does not reflect this reality and it tends to overestimate the unavailability as the process parameter changes.

Figure 1 Unavailability of the 1st Kind Action (PT-A Loop)

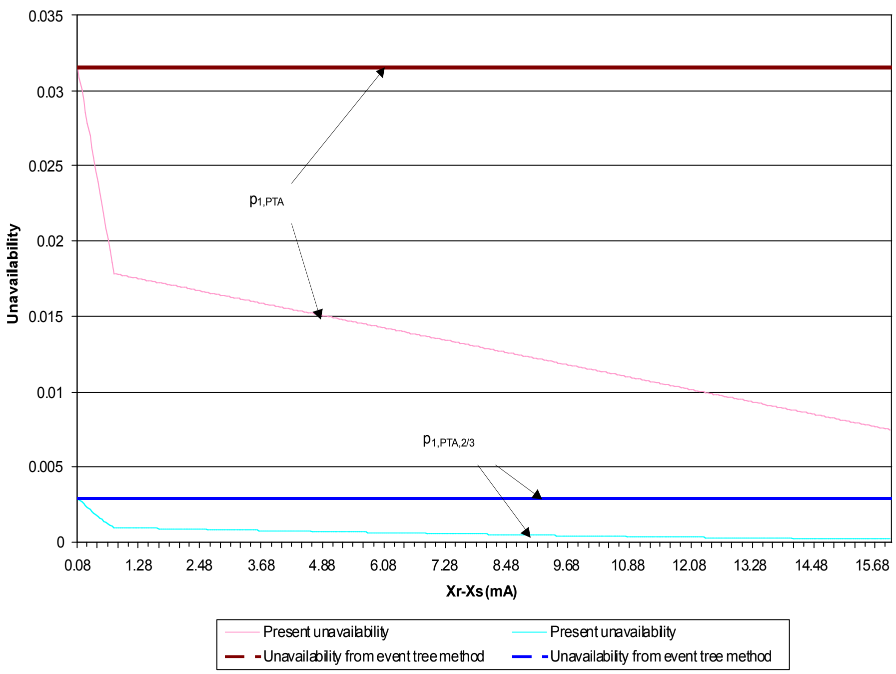


Figure 2 **Unavailability of the 2nd Kind Action (PT-A Loop)**

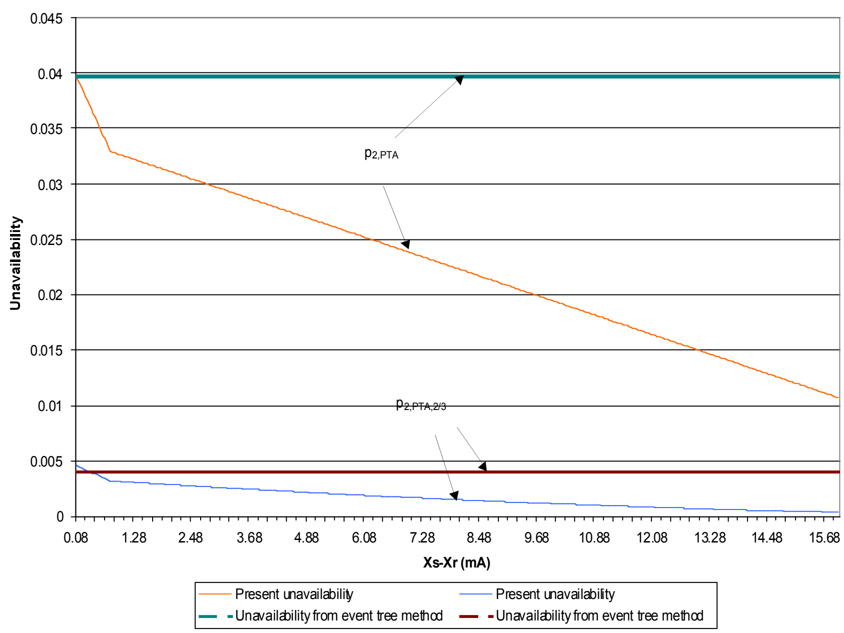


Figure 3 **Unavailability of the 1st Kind Action (DPT-A Loop)**

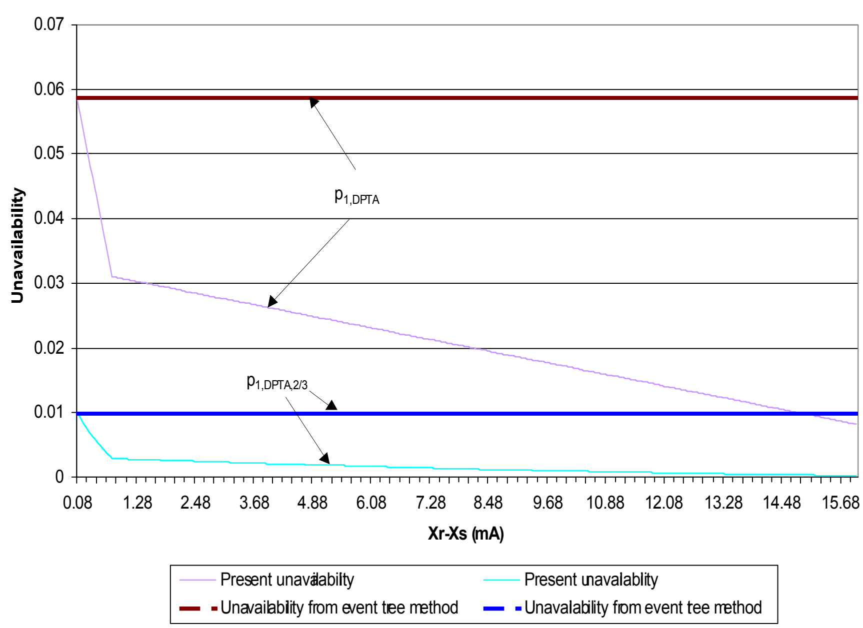


Figure 4 Unavailability of the 2nd Kind Action (DPT-A Loop)

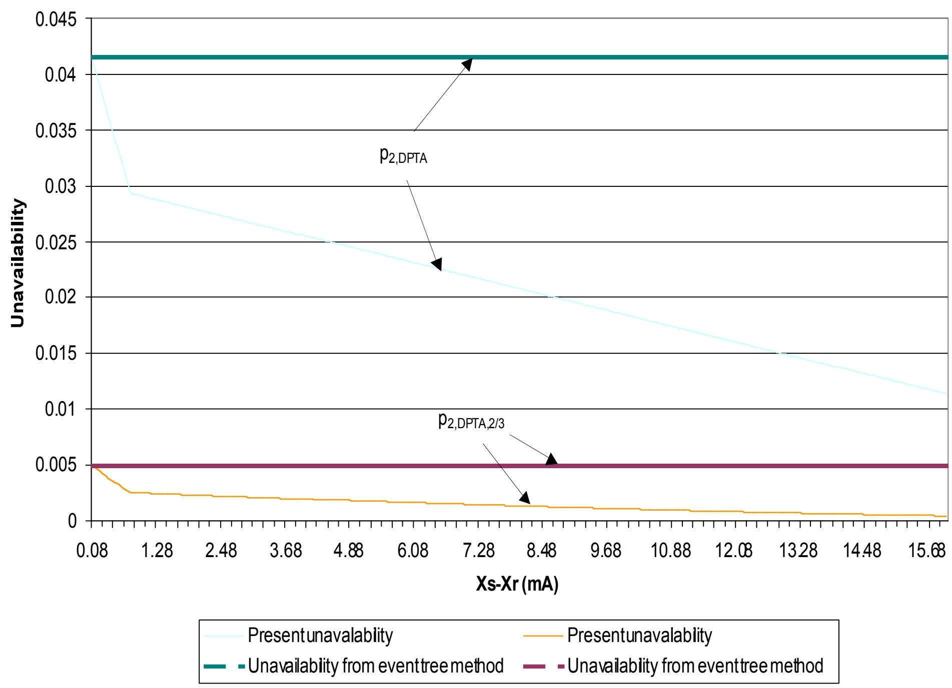


Figure 5 Unavailability of the 1st Kind Action (TT-A Loop)

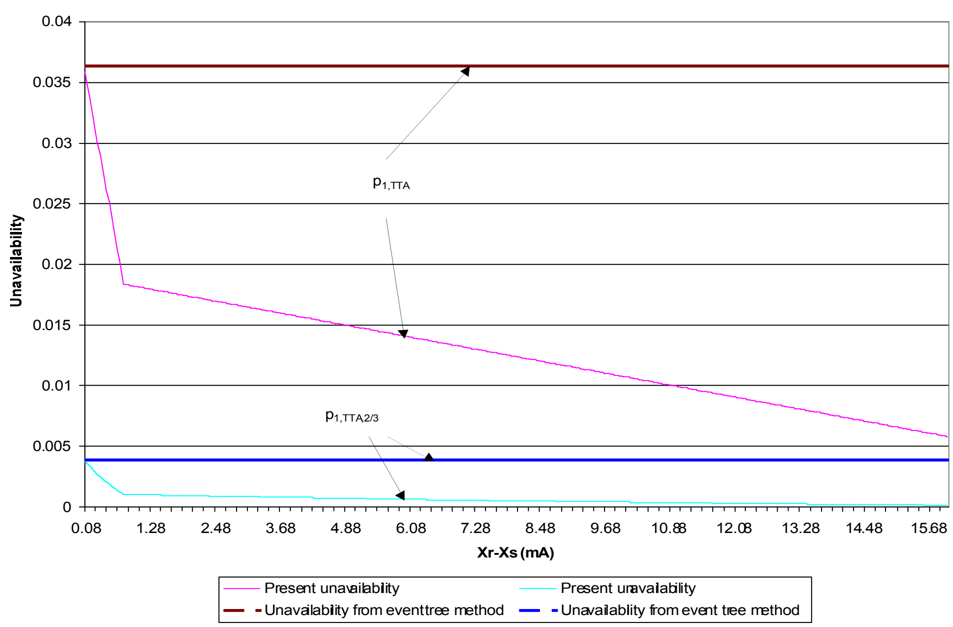
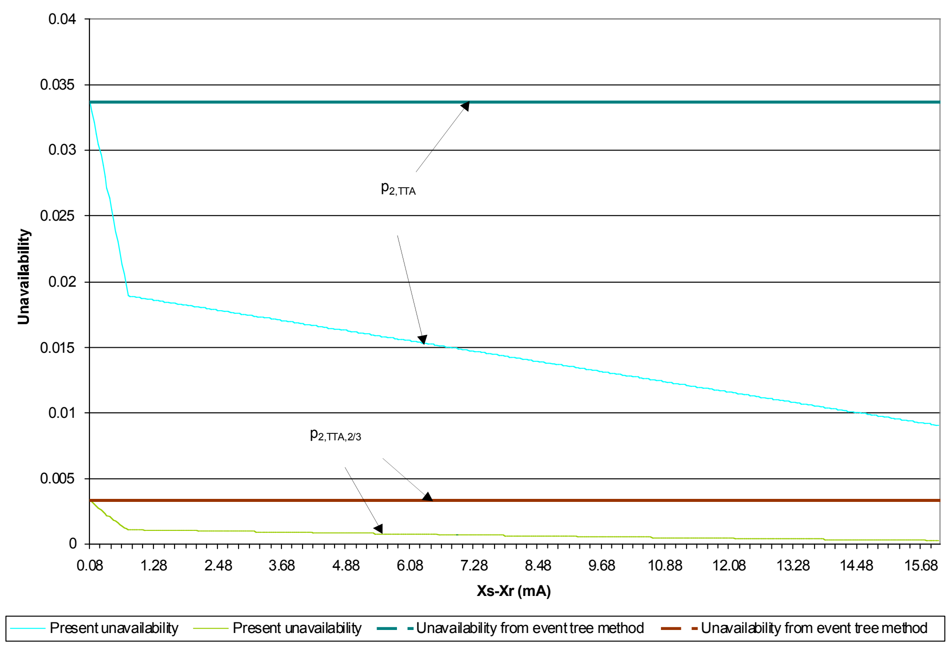


Figure 6 Unavailability of the 2nd Kind Action (TT-A Loop)



The delay time of an action depends on the rate of change of the process parameter. Unfortunately no test has been done, however, the response time of an instrument due to its drift can be calculated from Eq. (4) and (5) and compared to manufacturer's data. The response time of individual instrument and instrument loop are presented in TABLE 4.

Table 4 The delay time (seconds) of instrument and instrument loop due to drift

	First kind action		Second kind action	
PT	0.0565		0.1782	
DPT	0.1802		0.1561	
TT	0.0499		0.05	
A	0.1179		0.1458	
	Single vote	Majority vote	Single vote	Majority vote
PT-A loop	0.0565 < <0.1179	0.0032 < <0.3537	0.1458 < <0.1782	0.0213 < <0.5346
DPT-A loop	0.1179< <0.1802	0.014 < <0.5406	0.1458 < <0.1561	0.0213 < <0.4683
TT-A loop	0.0499< <0.1179	0.002 < <0.3537	0.0500 < < 0.1458	0.0025 < <0.4374

For pressure and differential pressure transmitter, the manufacturer estimated a response time of 0.2 seconds. The calculated response time for pressure and differential pressure transmitters due to drift are within the manufacturer's estimation. They are smaller than 0.2 seconds because other factors which affect the instrument response, such as random noise, characteristics of the electronic circuit, are not included in the calculation in the present work.

6. Conclusions

The effect of the instrumental drift on an action depends on the process parameter, thus the unavailability of the action due to drift is a function of the process parameter. Based on this fact, the following conclusions can be made, whenever the instrumental drifts are considered in the reliability analysis:

(1) An action may have different unavailability in different accident scenarios because the process parameter may vary in different fashion.
(2) Actions signaled by the same type of the process parameter at different location may have different unavailability, even if the actions are initiated by exactly the same type of instrument loop(s).
(3) To improve accuracy of the predictions, the unavailability and the process parameters should be calculated simultaneously.

7. Reference

[1] C.W. Gardiner: " Handbook of Stochastic Methods", 2nd Edt. Springer-Verlag (1985)
[2] A.F Jean etc. "Safety-related system observed component reliability data", IR-01530-01, Rev. 6, Point Lepreau Generating Station, 1998