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THE MEASUREMENT AND ANALYSIS OF THE DYNAMIC RESPONSE OF ALARM UNITS

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ABSTRACT

Alarm units are used to monitor some measured parameter (e.g., flux, pressure), and actuate an alarm and, if necessary, channel or reactor trips when abnormal values occur. If the measured parameter is above (or below for low alarms) the alarm setpoint, the output relay of the alarm unit opens. Alarm units are not simple comparators, they contain some type of internal amplifier driving a comparator, and the output relay has an inductive coil whose current cannot change instantly. It takes some time, called "response time", after the input signal passes the setpoint before the relay opens. In reactor safety analysis, it is important to allow for the dynamic response time of the instrumentation, including that of the alarm units. The ISA standard S67.06 defines methodologies which may be used to measure response times of the various components in the nuclear safety channel. The most popular tests are ramp tests and steps tests. This report reviews the theory of operation of alarm units and shows how to apply ramp tests and step tests to an alarm unit to obtain ramp response time, time constant and fixed delay.

This report also discusses how the time constant and fixed delay derived from step tests can be cross-checked against the ramp response time and how a first-order approximation to a higher-order system should be made.

Finally, the ramp and step response time measurement methods are applied to a test circuit representing a stand-alone alarm unit having known time constants and fixed delays. The results of these tests are evaluated as a means of assessing the practicality and accuracy of the test methods.

1. INTRODUCTION

Every CANDU reactor has two independent and diverse safety shutdown systems (SDS1 and SDS2) which act to mitigate potentially dangerous situations, in a safe manner by rapidly shutting down the reactor whenever operation outside the safe range is detected. SDS1 drops shutoff rods into the reactor, while SDS2 injects liquid poison into the moderator. Both SDSs are channelized and operate in a two-out-of-three logic mode. An SDS consists of a chain of elements, namely, primary sensors (transmitters), alarm detection, Boolean logic and the final mitigating device. Figure 1 shows a typical SDS chain. A major safety consideration is the speed with which the SDS acts. This speed consists of the response times of all the various elements in the SDS chain.

Alarm units are used to monitor some measured parameter (e.g., flux, pressure), and actuate an alarm and, if necessary, channel or reactor trips when abnormal values occur. The measured parameter is represented by a voltage or current input to the alarm unit. If the measured parameter is above[1] the alarm setpoint, the output relay of the alarm unit opens.

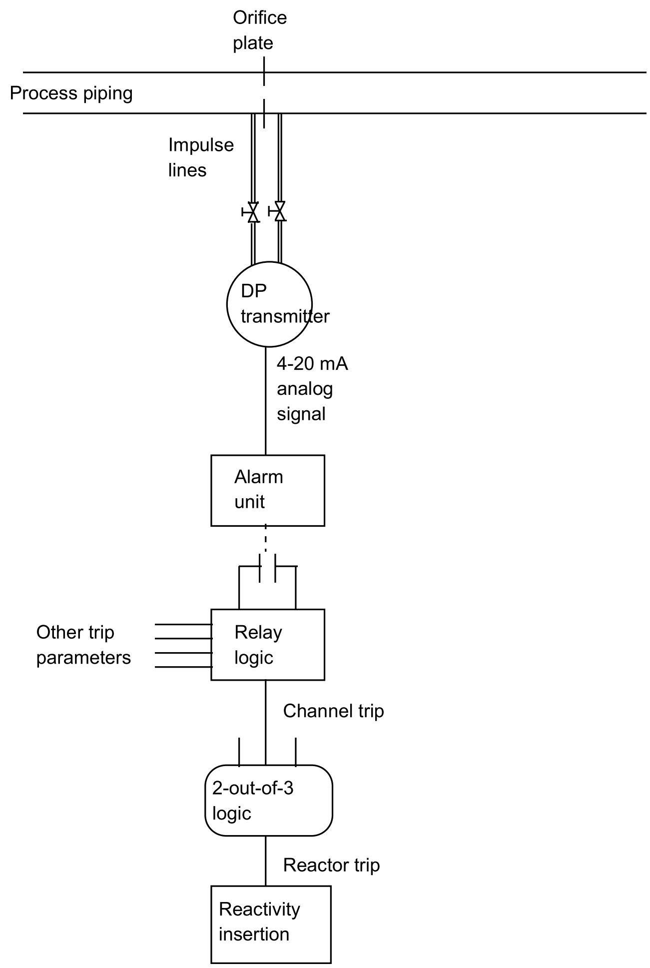

Figure 1: Typical CANDU Shutdown System chain

If the alarm unit functioned as a simple comparator, its output relay would open as soon as the input signal exceeded the setpoint. However, alarm units are not simple comparators. They contain some type of internal amplifier driving a comparator, and the output relay has an inductive coil whose current cannot change instantly. It takes some time, called "response time", after the input signal passes the setpoint before the relay opens.

In reactor safety analysis, it is important to allow for the dynamic response time of the instrumentation, including that of the alarm units, and the trip process. To obtain suitable values for the safety analysis, or confirm the validity of the values currently used, the dynamics of alarm

[1] For clarity in presentation, it is assumed in this report that the alarm occurs for high signals. However, all theory and measurement principles presented herein apply equally well to alarm units that alarm on low signals.

units should be measured. The ISA standard S67.04 [1] defines methodologies which may be used to measure response times and *response time characteristics*[2] of the various components in the nuclear safety channel.

There are several tests which could be applied, the most popular of which are ramp tests and steps tests [1]. This report reviews the theory of operation of alarm units and shows how to apply ramp tests and step tests to an alarm unit to obtain ramp response time, time constant and fixed delay. This report also discusses how the time constant and fixed delay derived from step tests can be cross-checked against the ramp response time, and how a first-order approximation to a higher-order system should be made. Finally, the ramp and step response time measurement methods are applied to a test circuit representing a stand-alone alarm unit having known time constants and fixed delays. The results of these tests are evaluated as a means of assessing the practicality and accuracy of the proposed test methods.

2. ALARM UNIT MODELS

Alarm units can be modelled in several ways. A fairly generalized model is shown in Figure 2. The dynamics of any signal conditioning amplifier can be represented by a transfer function *F(s)*, where *s* is the Laplace variable. This amplifier is then followed by a pure comparator which compares the amplifier output to the setpoint and which does not have any dynamics. The output of the comparator is a binary (on/off) signal, which drives the final relay coil. Because of the inductance of the coil and mechanical motion required to change state, the relay can be represented by a pure delay, which in Laplace notation is $e^{-s\tau}$.

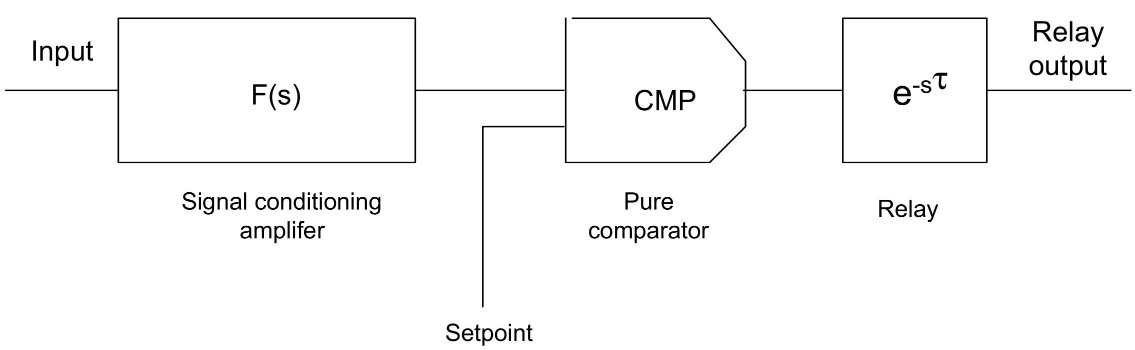

Figure 2: Model of a Typical Alarm Unit

The signal conditioning amplifier portion of the circuit for a typical alarm unit used in some CANDU reactors is shown in Figure 3. There are at least four capacitors (C104, C105, C108 and C109) which are likely to affect the dynamic response.

[2] Response time characteristics is defined in ISA S67.06 as: "Those properties (e.g., transfer function, time constant, delay time, power spectral density) of the equipment from which its response time can be determined."

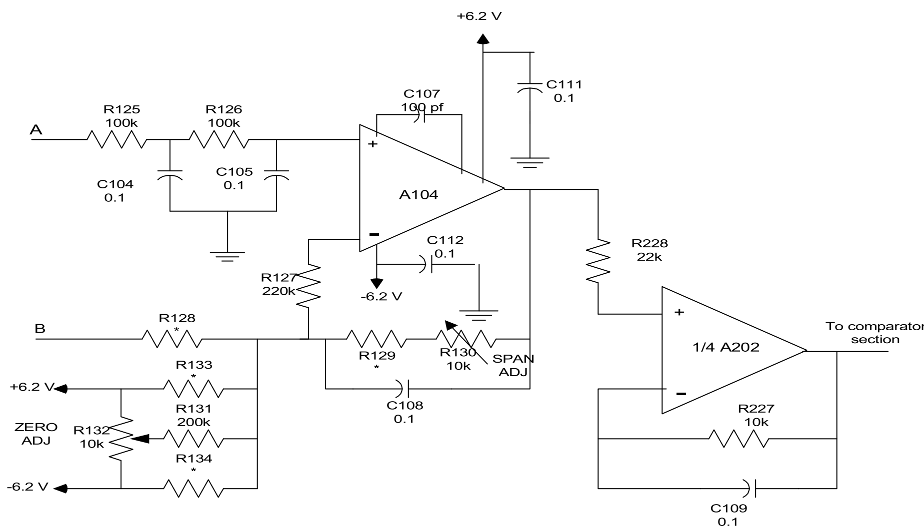

Figure 3: Partial Schematic Diagram of an Alarm Unit

3. RAMP RESPONSE THEORY

The response time of alarm units usually depends on the type of input waveform; they are not, in all cases, a property solely of the device. Assuming a first-order lag (simple RC circuit) for the amplifier portion of the alarm unit, the response time to a ramp input is given by:

$$\Delta T = t_2 - t_1 + \tau_d = \tau\left(1 - e^{-t_2/\tau}\right) + \tau_d \qquad (1)$$

where $t_1 = \dfrac{x_s - x_o}{c}$ and $t_2 = t_1 + \tau\left(1 - e^{-t_2/\tau}\right)$ (2)

where t_1 = time when the input crosses the trip setpoint
t_2 = time when the amplifier output crosses the trip setpoint
ΔT = response time of alarm unit
x_s = setpoint
x_o = initial value of input
c = ramp rate
τ = time constant of amplifier
τ_d = time delay associated with relay

Note from equation 2, that for ramps which are long ((x_s - x_o) large) and slow (c small), t_1 and t_2 can be large relative to τ. For t_1, $t_2 > 4\,\tau$, equation 1 simplifies, to better than 2% accuracy, to:

$$\Delta T = \tau + \tau_d \qquad (3)$$

For a generalized alarm unit having multiple time constants τ_i, provided $t_2 > 4\,\tau_{max}$ where τ_{max} is the largest of the τ_i time constants, it can be easily shown that:

$$\Delta T = \sum_i \tau_i + \tau_d \qquad (4)$$

The dynamic response of an alarm unit to a long slow ramp is illustrated in Figure 4(a). Note that the internal variable (amplifier output) is essentially linear and parallel to the ramp input when it crosses the trip setpoint at time t_2 and, from equation 2, $t_2 - t_1 = \tau$.

Also note that, for long slow ramps, ΔT, is independent of ramp length or ramp rate. Thus, every measurement gives the same ΔT and the two components of ΔT, τ and τ_d, cannot be determined independently from multiple measurements with slow ramps.

However, for shorter faster ramps where $t_1, t_2 < 4\tau$, equation 1 is a transcendental equation in t_2 and the solution depends on ramp length and ramp rate. Thus ΔT also depends on ramp length and ramp rate. The dependence of ΔT on ramp rate is illustrated by comparing Figure 4(a) to Figure 4(b). The fast ramp in Figure 4(b) results in a significantly smaller ΔT than that obtained with the slow ramp in Figure 4(a).

Note from Figure 4(b), that the internal variable (amplifier output) is slightly non-linear and is not parallel to the ramp input when it crosses the trip setpoint at time t_2 and $t_2 - t_1 < \tau$. Note also that, for two different known ramp rates, equation 1 can be solved to determine τ and τ_d. However, in practice this is difficult to do because of the sensitivity of the solution to ramp rate and setpoint, and because these equations must be solved numerically.

In summary, if the response time is to be determined from a ramp test, it is imperative that all conditions be known, or that the ramp be long enough and slow enough that $t_1, t_2 > 4\tau$. It is only in this latter condition that the ramp response time is a property solely of the device and not of the test conditions.

4. STEP RESPONSE THEORY

The above analysis applies to pure ramp inputs only. For arbitrary input waveforms, such as those which might occur for various safety analysis cases, the response times can be either larger or smaller than those obtained for ramp inputs. For example, for a simple first-order system, it is easily shown that the step response time is given by:

$$\Delta T = \tau_d + \tau \ln\left(\frac{x_f - x_o}{x_f - x_s}\right) \qquad (5)$$

where x_f is the final value of the step input and the other variables are defined as before.

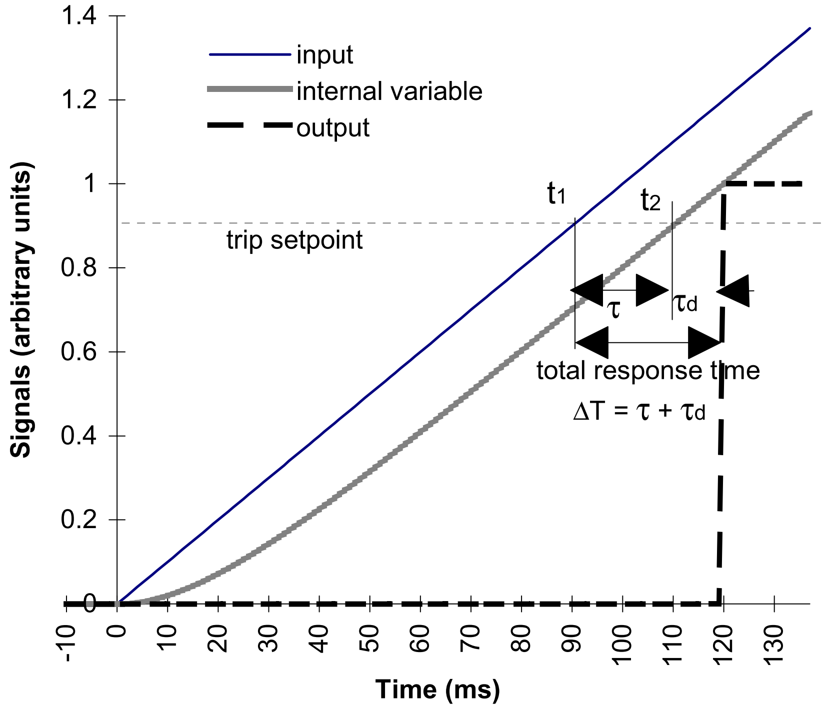

Figure 4(a): Response of an Alarm Unit with $\tau = 20$ ms and $\tau_d = 10$ ms to a Slow Ramp Total Response Time ΔT is 30 ms

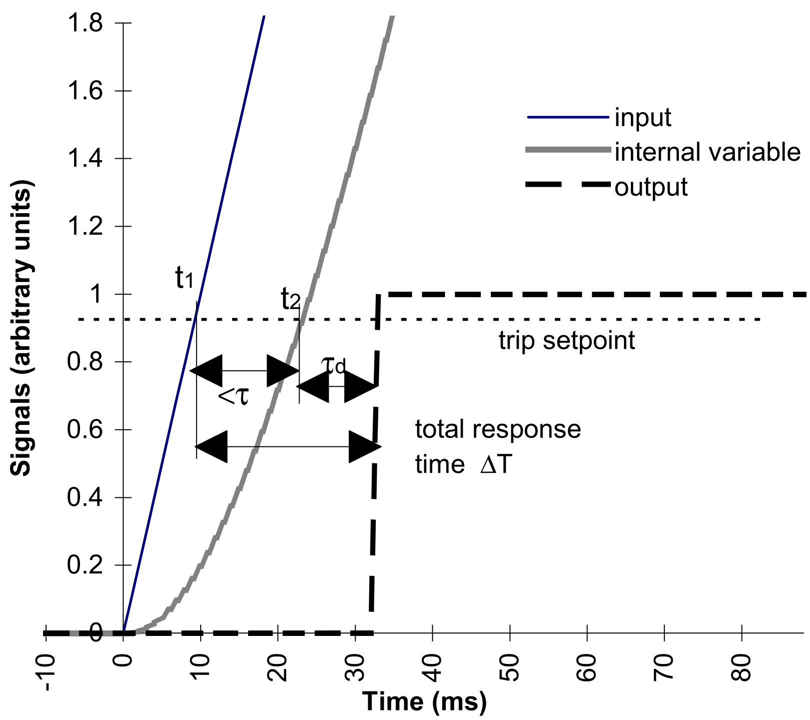

Figure 4(b): Response of an Alarm Unit with $\tau = 20$ ms and $\tau_d = 10$ ms to a Fast Ramp Total Response Time ΔT is 24 ms

Defining A as the normalized step amplitude, and B as a related parameter:

$$A = \frac{x_f - x_o}{x_s - x_o} \qquad \text{and} \qquad B = -\ln\left(1 - \frac{1}{A}\right) \tag{6}$$

equation 5 becomes

$$\Delta T = \tau_d - \tau \ln\left(1 - \frac{1}{A}\right) = \tau_d + B\,\tau \tag{7}$$

As is evident from equation 7, the step response time ΔT is a function of A, the normalized step amplitude which depends on:

- initial value of step input (x_o),
- final value of step input (x_f), and
- trip set point (x_s).

The step response is illustrated in Figure 5 for a first order system with τ = 20 ms and τ_d = 10 ms (the same system as illustrated in Figure 4). In Figure 5(a), the input step starts at 0 and exceeds the setpoint by only 5% (A = 1.05) resulting in a step response time of 68 ms. In Figure 5(b), the input step starts at 0 and exceeds the setpoint by 110% (A = 2.1) resulting in the much shorter step response time of 22 ms.

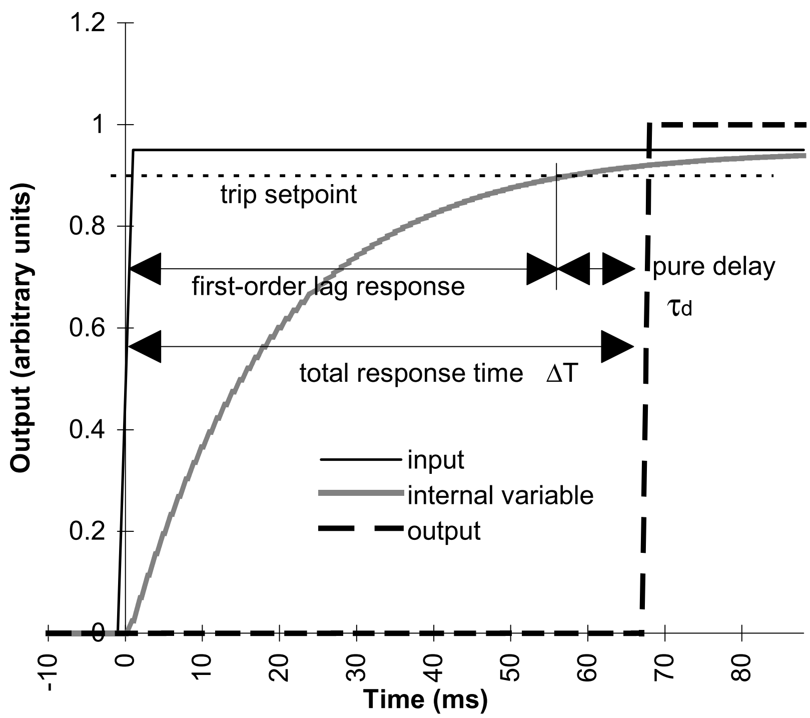

Figure 5(a): Response of an Alarm Unit with τ = 20 ms as τ_d = 10 ms to a Small Step Total Response Time ΔT is 68 ms

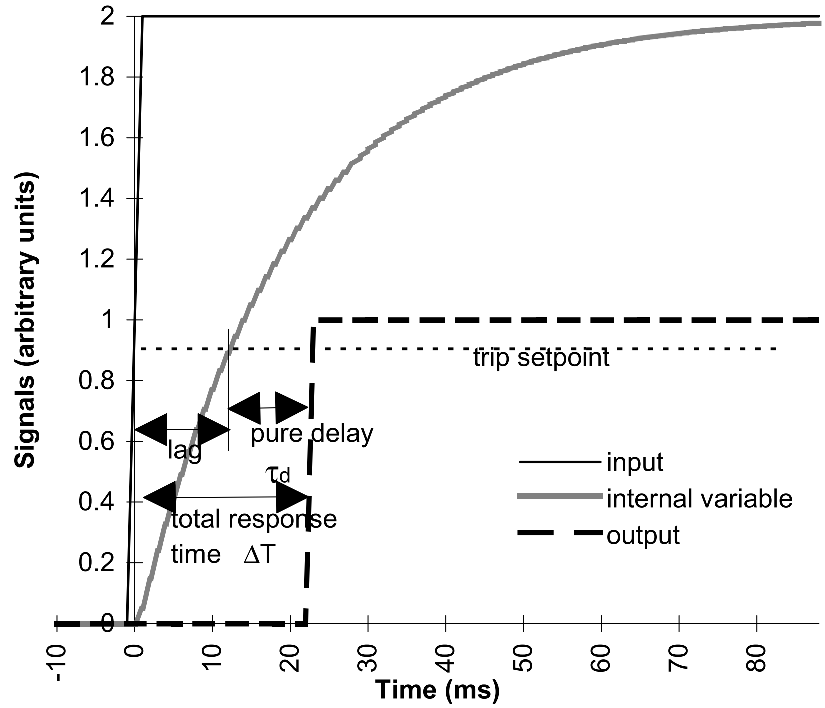

Figure 5(b): Response of an Alarm Unit with τ = 20 ms as τ_d = 10 ms to a Large Step Total Response Time ΔT is 22 ms

Step response time ΔT as a function of step amplitude A for a first-order system with τ = 20 ms and τ_d = 10 ms is illustrated as the heavy curve in Figure 6.

Higher-order models for the signal conditioning amplifier result in even more complex equations than equation 7 with several time constants. The step response time as a function of step amplitude for several systems of different orders but the same total slow-ramp response time (which, from equation 4, is just the sum of all time constants and all fixed delays even for higher-order systems) are shown in Figure 6. The higher-order models are created using multiple, equal time constants[3]. Note the differences in the first, second and third order curves, especially for large steps.

5. PROCEDURE FOR ASSESSING ALARM UNIT DYNAMIC RESPONSE

A measurement procedure for determining the most appropriate first-order τ and τ_d to use to represent a given alarm unit is outlined below. This procedure is applied and validated in the next section using first-, second- and third-order test circuits representing alarm units of known response characteristics.

The proposed procedure uses ramps (Step 1 of the procedure) and steps (Step 2 of the procedure) to obtain two independent measures of response and compares these as a consistency check (Step

[3] It is believed but not proven that the maximum difference between a high-order and a first-order system occurs if the time constants in a higher-order system are equal.

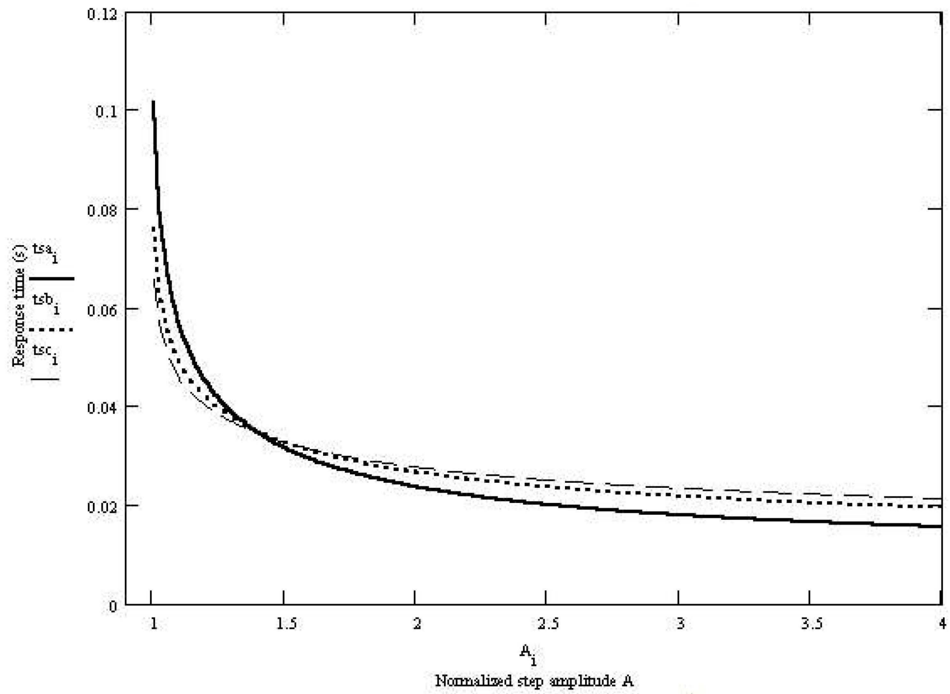

Figure 6: Step Response Times as Functions of Normalized Step Amplitude *A*
heavy line: first-order; one 20 ms time constant plus 10 ms delay
dotted line: second-order; two10 ms time constants plus 10 ms delay
dashed line: third-order; three 6.67 ms time constants plus 10 ms delay

3 of procedure). Steps 1, 2 and 3 are in compliance with Section 8 of the ISA standard for response time testing of nuclear safety related instrument channels in a nuclear power plant [1], which states that all test methods for response time measurements shall be validated by comparison with other direct methods in suitable laboratory or in-situ tests.

Analysis of a set of ramp measurements provides a means of obtaining the test estimate of the ramp response time, ΔT_{ramp}, to a long slow ramp. Also, ramp measurements provide an exceptionally accurate estimate of the trip setpoint, which is required to interpret the results of the step tests.

As noted in Section 4 the step response of a first-order system can be described by equation 7. Thus, for a first-order system, equation 7 can be solved explicitly for τ and τ_d by measuring two different step response times ΔT_1 and ΔT_2 in response to two different normalized step amplitudes A_1 and A_2. If more than two measurements are made then a least-squares approach can be used.

However, as discussed in Section 2, the amplifier portion of an alarm unit is usually not a first-order circuit but is typically second or third order. Consequently, equation 7 does not strictly apply. Furthermore, in practice, it is desirable to represent a higher-order circuit with a first-order model with τ_R and τ_{dR} selected to eliminate non-conservative error; i.e., eliminate underestimation of the step response time. A generalized scheme, based on the application of equation 7 to systems of any order, has been developed for selecting τ_R and τ_{dR} which will always return a τ_R and τ_{dR} that eliminates non-conservative error.

Once first-order τ_R and τ_{dR} have been selected to represent the higher-order system the consistency check as required by ISA 67.06 (Step 3 of the procedure) is straight forward. The sum ($\tau_R + \tau_{dR}$) will be approximately equal to $\sum_i \tau_i + \tau_d$ for the higher-order system, which, from equation 4 is just ΔT_{ramp} for a long slow ramp. Thus, consistency is proven if the difference between ($\tau_R + \tau_{dR}$) and ΔT_{ramp} is acceptably small, say less than 10%.

6. APPLICATION AND VALIDATION OF PROCEDURE

6.1 Test Circuit Description

A circuit with known characteristics was built, which is functionally equivalent to an alarm unit,. to test the theory and procedure. The circuit comprises a three-stage signal amplifier, a comparator and a relay. The fixed delay of the relay was independently measured and found to be $\tau_d = 44.2$ ms. The stages of the signal amplifier are each a first-order system with time constants of $\tau_1 = 30.5$ ms, $\tau_2 = 20.0$ ms and $\tau_3 = 14.7$ ms respectively. Switches provide the ability to switch stages in or out of the circuit. A first-order system consists of only stage one, a second-order system consists of stages one and two; a third-order system consists of all three stages. The slow ramp response times are thus theoretically (i.e., from circuit analysis) 74.7 , 94.7 and 109.4 ms respectively.

6.2 Application of Ramp Measurement (Step 1 of Procedure)

The application of step 1 of the procedure, use of ramps to measure setpoint and ramp response time of the first-order circuit, gives the results plotted in Figure 7. The setpoint x_s and ramp response time ΔT_{ramp} are found by fitting the voltage at the trip point x_T to a straight line:

$$x_T = \Delta T_{ramp}\, c + x_s \tag{8}$$

The measured ramp response times are 78.2, 99.3 and 114.8 ms for the first-, second- and third-order systems respectively. The differences with respect to the "theoretical" values derived from circuit analysis are all less than 5%, which validates the ramp measurements. The major cause of this difference is felt to be the measured relay delay time. If this relay delay time were truly 48 ms (cf. 44.2 ms), the differences would all be much smaller.

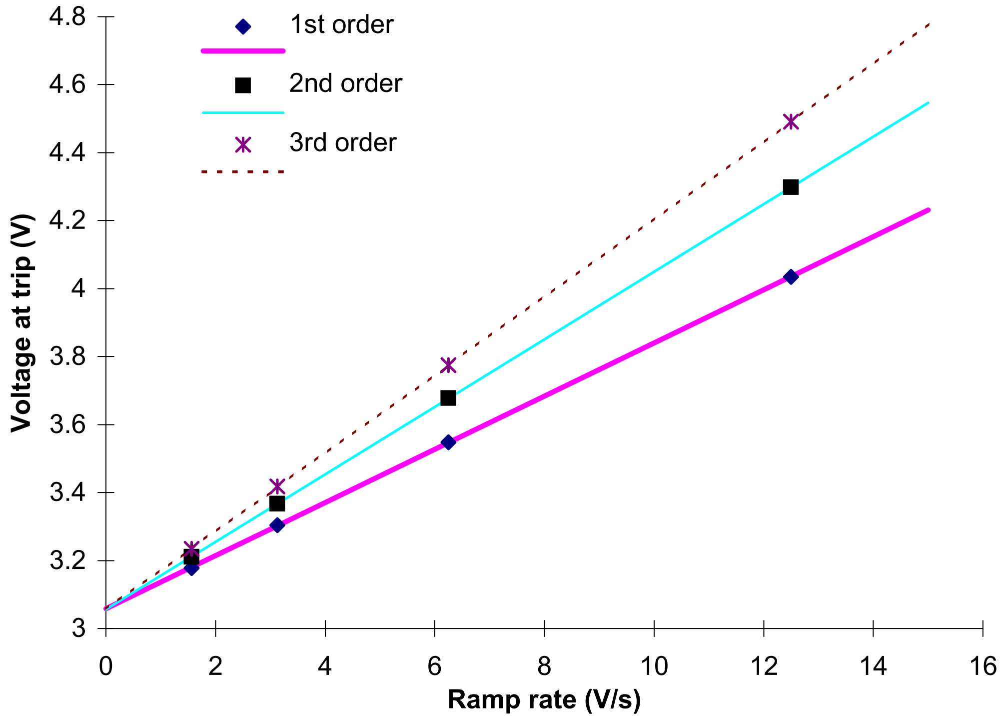

Figure 7: Ramp Voltage at the Moment the Relay changes State (x_T) as a Function of Ramp Rate (c) for 1st, 2nd and 3rd Order Circuits

6.3 Application of Step Measurement (Step 2 of Procedure)

Step 2 applies a set of steps of various amplitudes. The response times obtained are shown in Figure 8, and are analyzed to select representative first-order τ and τ_d to achieve minimum conservative response time error. The results are obtained by tangentially fitting a straight line

$$\Delta T_{stepR} = \tau_R B + \tau_{dR}$$

The values of τ_R and τ_{dR} obtained are 30.9 and 47.5 ms for the first-order circuit, 40.5 and 61.9 ms for the second-order circuit, and 43.5 and 75.4 ms for the third-order circuit.

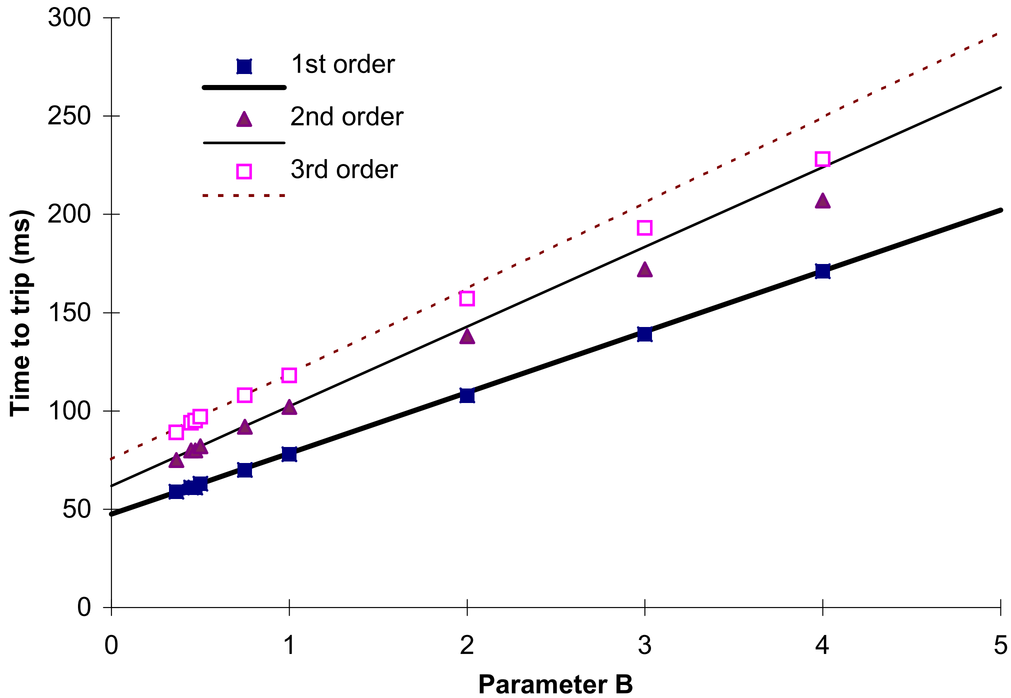

Figure 8: Step Response Time ΔT_{step} as a Function of B for test circuits.

The fitting errors are less than 2% for the first-order circuit. For the higher-order circuits, the fitting errors are also less than 2% in the critical range $0.4 < B < 1$, and less than 10% everywhere. The good fit of the first-order circuit is expected because theoretically the curve should be a straight line. For the higher-order circuits, the errors between the measured and fitted response times increase, as expected, for very small and very large values of B. However, in all cases, the error is less than 10% and is always positive (i.e., conservative).

Step 2 of the procedure requires use of x_s as determined from Step 1 of the procedure. Thus, the good agreement between ΔT_{stepR} and ΔT_{step} also verifies the portion of step 1 of the procedure used to determine setpoint and shows it to have sufficient accuracy.

6.4 Application of Consistency Check (Step 3 of Procedure)

The response time to a long slow ramp, ΔT_{ramp}, and $(\tau_R + \tau_{dR})$ as determined from step test are compared for the first-, second- and third-order circuits in Table 1.

Note that for each circuit the difference relative to ΔT_{ramp} is much less than 10% thus proving the consistency of the measurements as required by ISA 67.06. Also, the difference is always positive, indicating that the conservative tangential fit of the step test results in a total (ramp equivalent) delay which, as expected, is slightly greater than the measured ramp delay. This consistency check for the first-order circuit is extremely good (<1% difference) as expected.

Table 1: Comparison of $\tau_R + \tau_{dR}$ and ΔT_{ramp}

Circuit	τ_R ms	τ_{dR} ms	$\tau_R + \tau_{dR}$ ms	ΔT_{ramp} ms	error %
1st Order	30.9	47.5	78.4	78.2	+0.3
2nd Order	40.5	61.9	102.4	99.3	+3.1
3rd Order	43.5	75.4	118.9	114.8	+3.6

7. CONCLUSIONS

If an alarm unit were a pure comparator with a fixed delay the output relay would activate after a fixed time following the moment when the input voltage crosses the comparator setpoint. However, alarm units also comprise signal amplifiers which are characterized by one or more time constants. Consequently, the response time of the alarm unit depends on the shape of the input waveform.

This report provides theoretical analysis of alarm unit response to input ramps and input steps and formulates a simple method of obtaining a first-order representation of higher-order circuits that provides a conservative estimate of response time for all ramp and step inputs. This is a significant achievement since it provides a conservative alternative to the current non-conservative design and analysis practice of adding time constants of individual components to obtain a representative time constant for a higher-order system.

The procedure described herein supports a generalized approach to preparation of response time requirements for components. In particular, to specify response time in procurement documents the following statement should be used:

> "The response of the device to any input shall not be slower than that which would be obtained with a first-order device having a time constant of (*specify amount*) and a fixed delay of (*specify amount*). The manufacturer is required to conduct tests to verify this requirement is being met."

A detailed measurement procedure for assessing alarm unit dynamic response to ramps and steps and then for comparing results for consistency, as required in ISA Standard 67.06 Response Time Testing of Nuclear Safety-Related Instrument Channels in Nuclear Power Plants, has been developed. This procedure has been applied and validated using test circuits representing first-, second-, and third-order alarm units.

8. REFERENCES

[1] Instrument Society of America, "Response time Testing of Nuclear Safety-Related Instrument Channels in Nuclear Power Plants", ANSI/ISA-S67.06-1984.