AN EDDY VISCOSITY MODEL

FOR FLOW IN A TUBE BUNDLE

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ABSTRACT

The work described in this paper is part of the development of GENEPI, a 3-dimensional, finite element code, designed for the thermalhydraulic analysis of steam generators. It focuses on the implementation of two-phase flow turbulence-induced viscosity in a tube bundle.

The GENEPI code, as other industrial codes, uses the eddy viscosity concept introduced by Boussinesq for single phase flow. The concept assumes that the turbulent momentum transfer is similar to the viscous shear stresses. Eddy viscosity formulation is reasonably well known for single phase flows, especially in simple geometries (i.e., in smooth tube, around a single body, or behind a row of bars/tubes), but there exists very little information on it for two-phase flows.

An analogy between single and two-phases is used to set up a model for eddy viscosity. The eddy viscosity model examined in this paper is used for a tube bundle geometry and, therefore, is extended to include anisotropy to the classic model. Each of the main flow directions (cross flow inline, cross flow staggered, and parallel flows) gives rise to a specific eddy viscosity formula. The results from a parametric study indicate that the eddy viscosity in the staggered flow is roughly 1.5 times as large as that for the inline cross flow, 60 times as large as that for the parallel flow, and 10^5 as large as that for the molecular viscosity. Then, the different terms are combined with each other to result in a global eddy viscosity model for a steam generator tube bundle flow.

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NOMENCLATURE

a constant (= 0.047)
D diameter (m)
D_H hydraulic diameter (m)
G mass flux (kg m^{-2} s^{-1})
g gravitational acceleration (m s^{-2})
h heat transfer coefficient (W m^{-2} K^{-1})
H enthalpy per unit mass (J kg^{-1})
K function of m
L latent heat of vaporisation (J kg^{-1})
m tube diameter to pitch ratio
P pressure (Pa)
P pitch (m)
Q mass flow rate (kg s^{-1})
R pipe radius (m)
Re Reynolds number
t time (s)
T_p primary side temperature (K)
T_w secondary side wall temperature (K)
u^* wall friction velocity (m s^{-1})
v, V velocity (m s^{-1})
x quality $=\dfrac{H - H_L^{sat}}{L}$
y cross coordinate (m)

Greek letters

α void fraction
β porosity
γ_0 heat area density (m^2 m^{-3})
$\underline{\underline{\Lambda}}$ solid fluid friction tensor (s^{-1})
μ molecular dynamic viscosity (kg m^{-1} s^{-1})
μ_T turbulent dynamic viscosity (kg m^{-1} s^{-1})
ν kinematic viscosity (m^2 s^{-1})
ρ density (kg m^{-3})
σ shear stress (kg m^{-1} s^{-2})
σ_0 wall shear stress (kg m^{-1} s^{-2})
τ_b bundle residence fraction
χ_T turbulent enthalpy diffusion (kg m^{-1} s^{-1})

Superscript

' fluctuations
A axial (parallel)
C cross flow
L line
S staggered

Subscript

b bundle
G gas
i interaction
L liquid
R relative
T turbulent

1. INTRODUCTION

GENEPI is a 3D finite element code, designed for the analysis of Steam Generator (SG) thermalhydraulics. Its purpose is to predict the secondary fluid flow inside a SG, under operating or incidental conditions, in order to assess the heat exchanger performances (steam flow rate, heat rate, recirculation flow rate) and to describe the flow pattern (bubble size, void fraction profile) and the flow dynamics (liquid and gas velocities) necessary for a vibration analysis. Designed for industrial SGs, GENEPI is able to predict global flow rates and detailed distributions of various parameters, which can be used for vibration analyses. In pressurized water reactor SGs, the tube bundle offers strong anisotropy to the flow and GENEPI code

incorporates that in its models. This paper is focused on two-phase flow turbulence-induced momentum diffusion, or commonly called eddy shear stress.
The first part describes the equations governing two-phase flow and the physical model implemented in GENEPI for turbulent shear stress tensor; the second part shows the influence of various eddy viscosity models on the flow in a tube bundle.

2. THREE DIMENSIONAL TWO-PHASE FLOW MODEL

In steam generators, the secondary fluid is a mixture of two phases (steam/water), and under operating conditions, the flow pattern is mostly a bubbly, dispersed type of flow. We present hereafter the equations commonly used to predict two-phase flows, based on the Eulerian approach.

2.1. Governing equations

Based on the conservation of mass, momentum and energy, the flow is described by a set of six local instantaneous phase equations and three jump conditions on interface area. Due to random nature of the flow, the equations are time-averaged. The equations for each phase are added together; thus, they are reduced to three time-averaged equations for a liquid-gas, homogeneous mixture. Due to the complexity of the tube bundle, they are also space-averaged over a control volume.
This averaging process leads to a new set of three time and space-averaged governing equations which govern the behaviour of the mean flow in an equivalent fluid-solid porous continuum. The fluid volume fraction or porosity is defined by :

$$\beta = \frac{\text{volume occuped by steam and water}}{\text{total volume}}$$

The mixture equations make the interfacial terms disappeared, but they include the drift terms.
The following assumptions are applied :

- surface tension, work of interaction forces, and viscous and turbulent dissipation are ignored,
- thermodynamic saturation, which entails : $P_G = P_L$,
- pressure terms $\frac{\partial P}{\partial t} + \vec{j} \cdot \overline{\text{grad}}\, P$ are negligible in the energy equation,

The three governing equations, in Cartesian coordinates, for the mixture are as follows :

Continuity equation :

$$\beta \frac{\partial \rho}{\partial t} + \text{div}\left(\beta\, \rho\, \vec{V}\right) = 0 \tag{1}$$

Momentum equation :

$$\beta\rho\frac{\partial\vec{V}}{\partial t}+\beta\rho\left(\vec{V}\cdot\overline{\text{grad}}\right)\vec{V}-\overline{\text{div}}\left(\beta\,\mu_T\left(\overline{\overline{\text{grad}}}\,\vec{V}+\overline{\overline{\text{grad}}}^{T}\,\vec{V}\right)\right)+\beta\overline{\text{grad}}\,P=$$
$$\beta\rho\vec{g}-\beta\overline{\overline{\Lambda}}\,\rho\vec{V}-\overline{\text{div}}\left(\beta x(1-x)\rho\vec{V}_R\oplus\vec{V}_R\right) \tag{2}$$

Enthalpy equation :

$$\beta\rho\frac{\partial H}{\partial t}+\beta\rho\left(\vec{V}\cdot\overline{\text{grad}}\,H\right)-\text{div}\left(\beta\,\chi_T\,\overline{\text{grad}}\,H\right)=\tau_b\gamma_0 h(T_P-T_w)-\text{div}\left(\beta x\,(1-x)\,\rho L\,\vec{V}_R\right) \tag{3}$$

with closure relations :

- density ρ and quality x are functions of pressure P and enthalpy H,
- latent heat L is a function of pressure P,
- χ_T, μ_T, $\vec{V}_R=\vec{V}_G-\vec{V}_L$, $\overline{\overline{\Lambda}}$ are obtained by empirical correlations,
- T_P is the solution of the energy equation solved on the primary domain and h is given by heat transfer model between the primary and the secondary fluids through the tubes,

and :

- β, τ_b and γ_0 are properties of SG internal structures.

The diffusion terms in the momentum equation model the mixture turbulent stress tensor, similar to the Reynolds stress tensor in single phase. The model is based on the virtual kinematic viscosity concept, also called the eddy viscosity concept, for which the following assumptions are made :

- diffusion flux is proportional to the mean gradient,
- momentum is carried from high level turbulence areas to lower-level turbulence areas.

Hence :

$$-\rho<\overline{v'_i\,v'_j}> \;=\mu_{T_i}\,\frac{\partial V_i}{\partial x_j}\,,\quad\text{with}\quad\mu_{T_i}\geq 0 \tag{4}$$

Given the symmetry condition for the stress tensor (issued from the angular momentum conservation equation), it is customary to write :

$$-\rho<\overline{v'_i\,v'_j}> \;=\mu_T\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right) \tag{5}$$

as mentioned in equation (2).

Currently, GENEPI solves steady state cases and its algorithm is based on a semi-implicit scheme. Its final result is obtained from a converged solution of a transient case.

2.2 Eddy viscosity model

The eddy viscosity occurring in the turbulent momentum transfer model described above is usually reduced to a scalar, which is a good approximation for isotropic turbulence. A new model is proposed in this paper to take into account the 3-D features of the flow using the eddy viscosity concept.

Different literature models are described first.

* Schlichting Model

Based upon the Prandtl mixing length theory, a correlation is derived from a study of the velocity profiles in the wake behind a circular cylinder [2], which leads to :

$$\mu_T = a\, G_{max}\, L \tag{6}$$

where :

- $a = 0{,}047$ as Schlichting coefficient,
- G_{max} : maximum mass flux in the direction of flow,
- L : a function of the mixing length.

Following theoretical and experimental investigations, R. Gran Olson [2] generalized the model for a wake behind a row of bars :

$$\mu_T = K(m) \cdot \left[G_{max} - G_{min}\right] \cdot P \tag{7}$$

where :

- P is the bundle pitch,
- m is the ratio of the tube diameter over the pitch P,
- K is a function varying with m,
- G_{min} : minimum mass flux in the direction of flow.

Goertler [2] found, for instance, $K = 0.033$ for $m = \frac{1}{8}$.

J.G. Van Bohl [2], performing experiments on several rows of parallel polygonal bars, has determined a limit ratio above which the jets between rows are closing in, thus increasing momentum diffusion. On the basis of experiments, the K function increasing with m can be assessed.

* Blasius-Nikuradze Model

This model has been set up for turbulent flows in smooth pipes.

- For Reynolds number bounded by 2000 and 10^5, Blasius empirical correlation established for the assessment of head losses along a smooth cylinder pipes allows modelling the wall friction velocity by :

$$u^* = \sqrt{\frac{\sigma_0}{\rho}} = \sqrt{\frac{\lambda}{8}}\, V \tag{8}$$

$$\lambda = \frac{0{,}316}{Re^{1/4}} \tag{9}$$

- Based on Prandtl's hypothesis, the turbulent stress tensor is expressed as :

$$\sigma = \mu_T \frac{dv}{dy} \tag{10}$$

- Assuming a linear shear stress distribution over the tube diameter of R radius :

$$\sigma = \sigma_0 \left(1 - \frac{y}{R}\right) \tag{11}$$

where :

σ_0 is the wall shear stress.

- With equation (8) to (11) Nikuradze [2] determined the variation of $\frac{\mu_T}{u^* R}$ over pipe diameter, from a measured velocity distribution.

In GENEPI, as flow calculation is performed for each control volume, we have decided to consider the average eddy viscosity. So for an axial flow, we find :

$$\mu_T = 0{,}0053\ \mu\ (Re)^{1-\frac{1}{8}} \tag{12}$$

* Van Der Welle Model

This model takes into account the two phase features. Van Der Welle [3] assumes that the turbulent shear stress may be subdivided into a part due to the movement of the liquid phase (μ_{T_L}) and another part attributed to the momentum exchange caused by the presence of the gas phase (phase interaction μ_{T_i}), so :

$$\mu_T = (1-\alpha)\left(\mu_{T_L} + \mu_{T_i}\right) \tag{13}$$

With μ_{T_L} and μ_{T_i} resulting from empirical correlations :

$$\mu_{T_L} = \mu_L \sqrt{\frac{\lambda}{8}}\ \frac{Re_L}{30} \tag{14}$$

$$\lambda = 0{,}0056 + \frac{0{,}5}{Re_L^{0{,}32}} \tag{15}$$

$$Re_L = \frac{(1-\alpha)\ \rho_L\ V_L\ D}{\mu_L} \tag{16}$$

$$\mu_{T_i} = \alpha\ \mu_L \left[100 + 0{,}0024\ \frac{V_G\ \rho_L\ D}{\mu_L}\right] \tag{17}$$

* Dwyer Model

This model has been implemented in the TRIO-VF computer code, a thermalhydraulic code for transient 3D turbulent single phase flows [4]:

$$\mu_T = \mu\ \left(T_X + T_Y + T_Z\right) \tag{18}$$

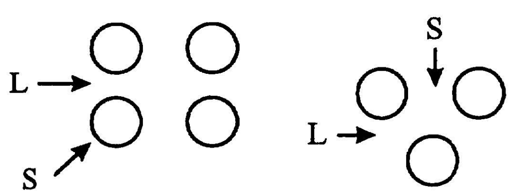
$$
\begin{aligned}
&T_X = 2{,}98 \;\; 10^{-5}\, Re_X^{1{,}71} && \text{if} \quad Re_X > 450 \\
&T_Y = 2{,}98 \;\; 10^{-5}\, Re_Y^{1{,}71} && \text{if} \quad Re_Y > 450 \qquad (19) \\
&T_Z = 10^{-3}\, Re_Z^{1{,}71} && \text{if} \quad Re_Z > 2500 \\
&T_X = T_Y = T_Z = 0 && \text{otherwise,}
\end{aligned}
$$

where Re_X, Re_Y, Re_Z are the Reynolds numbers computed in each direction (X, Y for cross flow directions and Z for axial direction).

Figures 1 and 2 show the eddy viscosity plotted against mass flux and void fraction, respectively, for a Freon R114 two-phase flow, assuming a slip ratio between gas and liquid velocity of 2.
Profiles are similar, except at very high void fraction ($\alpha \approx 1$), for which only Van Der Welle Model drops.
In general, the correlations set up for cross flow (the Schlichting and Dwyer correlations for cross flow) result in eddy viscosity 30 times higher than the correlations established for turbulent flow through smooth pipes (Blasius-Nikuradze & Van Der Welle & Dwyer axial). Thus, models are consistent with each other.

2.3 3D Implementation

In a SG tube bundle, there are 3 main flow configurations :



- axial flow, where an analogy between the flow through smooth pipes and the axial flow in a subchannel is made,
- inline cross flow,
- staggered cross flow.

We propose that the eddy viscosity be a function of 3 terms dependent on turbulent diffusion in each direction :

$$\mu_T = \phi\left(\mu_T^A , \mu_T^L , \mu_T^S\right) \qquad (20)$$

Arbitrary, due to the lack of analytical experiments, we propose a quadratic average between the axial viscosity and the cross viscosity, that allows satisfying the limit cases :

$$\mu_T = \sqrt{\left(\mu_T^A\right)^2 + \left(\mu_T^C\right)^2} \qquad (21)$$

The cross component is issued from an interpolation between inline and staggered models, according to a cosines law, as used for the friction factor in the CAFCA code [5] and in GENEPI [1]:

$$\mu_T^C = \frac{1}{2}\left[\left(\mu_T^L + \mu_T^S\right) + \left(\mu_T^L - \mu_T^C\right)\cos 2k\,\psi\right] \qquad (22)$$

with : $\psi = \left(\vec{G}, \vec{G}^C\right)$, G^C is the cross component of the mass flux,

and : $k = 2$ for square pitch bundle,
$k = 3$ for triangular pitch bundle.

Each terms in equations (21) and (22) is calculated with models described in section 2.2, by using the characteristic sizes for each direction (velocity, wake width) shown in the table 1 below.

	Square bundle			Triangular bundle		
	Axial	Line	Staggered	Axial	Line	Staggered
Mass Flux	G^A	G^C	G^C	G^A	G^C	G^C
Characteristic Length (L in Schlichting model, D in Reynolds numbers)	D_H	P	$\sqrt{2}\,P$	D_H	P	$\sqrt{3}\,P$

Table 1

Turbulent diffusion in the energy equation (3) is deduced from the kinematic viscosity and turbulent Prandtl number :

$$Pr_T = \frac{\chi_T}{\mu_T} \tag{23}$$

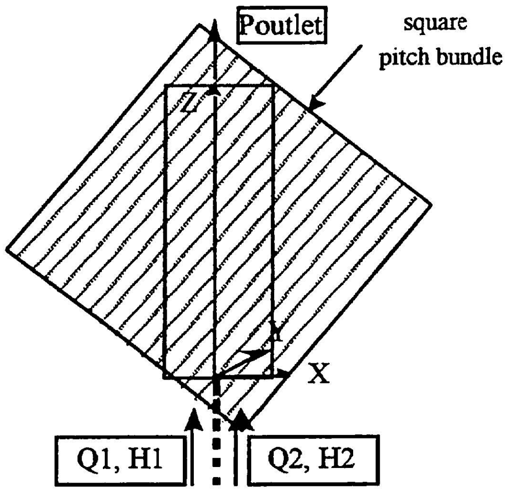
It is customary to consider a turbulent Prandtl number around unity in a subchannel [13].

As a result, turbulent diffusion in terms of momentum and enthalpy transfer is non-uniform in the tube bundle, strongly dependent upon the inclination of the flow towards the bundle obstacle.

3 APPLICATION TO AN INLET ENTHALPY NON-EQUILIBRIUM FLOW IN TUBE BUNDLE

The test section is a vertical channel of rectangular cross-section (0.411 × 0.0685 m), about 1 m high, fully occupied by a tube square pitch bundle (pitch P = 0.01370 m, tube diameter D = 0.00952 m). An enthalpy step as inlet boundary condition is imposed to the vertical upward flow (the inlet section is divided into two areas : in half of the section, the fluid - Freon R114 - comes with a constant enthalpy H1, in the other, the fluid - also Freon R114 - comes with a constant enthalpy H2 different from H1) . Outlet pressure is set to 3.86 10^5 Pa. There is no energy source term, so no boiling correlation is required. Three geometries are examined : vertical bundle (axial flow), oblique bundle (the angle of flow inclination with regard to the tube axis = 30°), and horizontal bundle (staggered cross flow). Enthalpy diffusion induced by turbulence is studied for a liquid (calc.1) and a two phase flow (calc.2).

The GENEPI code is used to solve the Navier Stokes equations for the mixture, with the following assumptions.



- Kinematic equilibrium exists between phases : $V_G = V_L$.
- Turbulent viscosity : the Schlichting model for cross flows (a = 0.047), the Blasius Nikuradze model for parallel flows, velocities projected onto parallel and cross directions in the bundle local system of reference.
- Turbulent Prandtl number = 1.
- Friction due to tube bundle : the Colburn model [10] for parallel flow, the Idelcik model [11] for cross flow, the Chisholm correlation [12] for the two-phase multiplier.

The table 2 below summarises the boundary conditions of the six simulations performed with GENEPI and the associated void fractions, calculated from enthalpies.

	Bundle position
CASE a	Vertical bundle ⇒ axial flow
CASE b	Bundle turned by 60° from vertical ⇒inline cross flow + parallel cross flow
CASE c	Horizontal Bundle turned by 45° from Ox ⇒ staggered cross flow

		Boundary conditions					Void fraction (%)		
		Q1(kg/s)	Q2(kg/s)	H1(J/kg) *T1(°K)*	H2(J/kg) *T2(°K)*	P_{outlet} (10^5 Pa)	α_1	α_2	α_{outlet}
Calc.1	CASE a CASE b CASE c	} 10.	} 10.	}0.6 10^5 *297°K*	}0.7 10^5 *307°K*	} 3.86	} 0.	} 0.	} 0.
Calc.2	CASE a	10.	10.	0,83 *318,9°K*	0,90 *318,8°K*	3.86	0,24	0,76	0,57
	CASE b	10.	10	1,0 *332,6°K*	1,1 *335,2°K*	3.86	0,20	0,76	0,91
	CASE c	10.	10.	0,95 *329°K*	1,05 *330°K*	3.86	0,34	0,78	0,90

Table 2

In order to emphasise the diffusion cone along the test section, we have represented, in Figures 3 and 4, the enthalpy profiles for two heights z = 0.1 m and z = 0.8 m, measured from the inlet face. We notice, as expected, that the more the bundle is inclined towards the flow (case a to case c) thus constituting a bigger obstacle to the flow, the more the turbulent diffusion increases. The difference between the first and the second calculation is due to the mixture nature, either liquid or liquid-gas, which changes the parameters in the model (density, molecular viscosity).

4 DISCUSSION-CONCLUSION

This article draws a list of current eddy viscosity models, established for one-dimensional or two-dimensional axisymetric flows. For more complex flow, such as occurs in steam generators tube bundle, we propose to split the turbulent viscosity into three terms, correlated to the three main directions of the flow (parallel flow, inline cross flow, and staggered cross flow), then to use the most appropriate model in each direction, and finally to combine between each other the three quantities. The goal purchased is to vary the diffusion of momentum and enthalpy according to the obstacle encountered by the flow. The simulation test illustrates this effect.

However, the eddy viscosity concept is a very approximate way to assess turbulent diffusion, which hardly takes into account turbulence dynamic phenomena such as production or dissipation. Above all, most models have been established for single phase flow whereas flows occurring in steam generators are two-phase and many papers e.g. [6,9], indicate that two-phase turbulence greatly depends on the flow pattern, void fraction, bubbles size, liquid velocity. So, we are conscious that the proposed model is very approximative.

As a result, experimental validation is required before going further on modelling, which could start first with a turbulent single phase flow under different bundle geometries, then on the impact of bubbles.

ACKNOWLEDGEMENTS

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Figure 1 : EDDY VISCOSITY MODEL

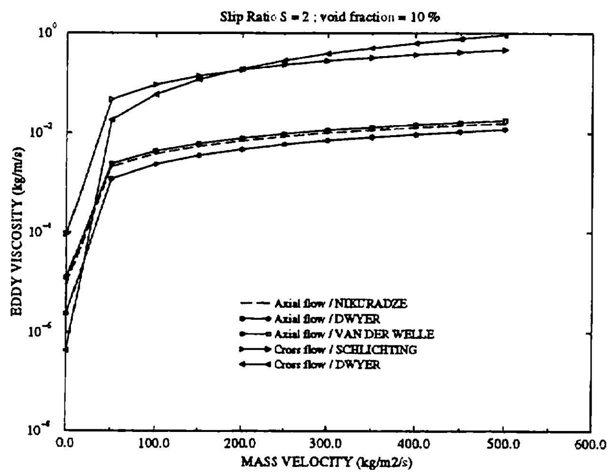

Figure 2 : EDDY VISCOSITY MODEL

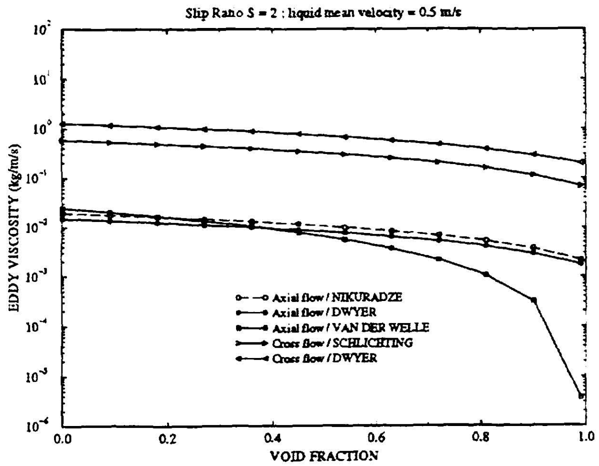

Figure 3 : ENTHALPY PROFILES
Two–phase flow mixture

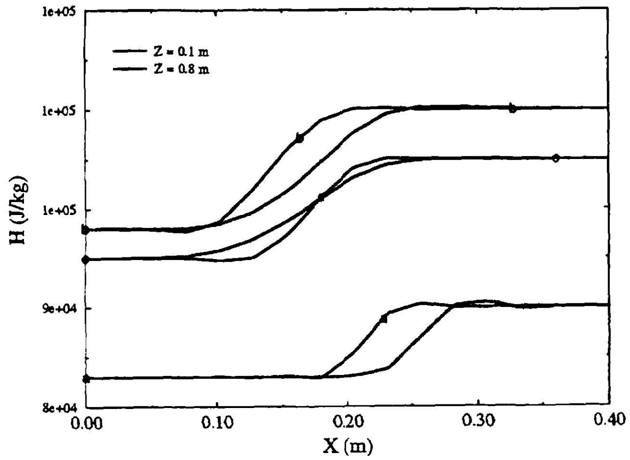

Figure 4 : ENTHALPY PROFILES
Liquid single–phase flow