Limit Analysis of Pressure Components based on Repeated Elastic Analyses

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Abstract

Simplified methods of limit load determination, based on repeated linear elastic finite element analysis, have been in existence for over a decade. Considering the acceptable quality of results and cost effectiveness, these methods are emerging as reliable alternatives for the more involved inelastic finite element analysis. However, application of these methods to three-dimensional structures has been limited. A cylindrical pressure vessel with an oblique nozzle is considered in this paper to demonstrate that the method works well for three-dimensional structures. The reduced modulus method is then extended for determining the limit loads of orthotropic components. Although the Hill's yield criterion is considered, the proposed method is generic.

1. Introduction

An endeavor to develop pressure component designs that are economically and functionally viable has resulted in progressive detour from conventional analytical methods of analysis. Conventional design methods had evolved using simplifying assumptions, such as 'beam on elastic foundation concepts,' during a time when computer based numerical analyses were in their infancy. The physical feel of the problem offered by the classical methods is offset by their lack of generality and inbuilt over-conservatism, thereby leading to expensive designs. Although a thicker vessel may be structurally safer, at elevated temperatures the thermal stresses developed become a matter of concern. The stress classification procedures, which conventional methods heavily rely on, are cumbersome and in some cases debatable. There are other problems with conventional analysis methods which could lead to designs where one location of a component would end up becoming more conservative than some other location. This redundant conservatism does not add to the gross structural safety, but only increases the gross weight of the component. An optimum design is one where over-conservatism is avoided at every location of the structure by adopting methods that are capable of providing consistently accurate results.

With advances in computing facilities and ready availability of general purpose finite element softwares, linear elastic finite element analysis is replacing many of the traditionally used methods. However, linear elastic finite element analysis based methods, involving stress categorization, are also over conservative. This has motivated designers to explore the possibility of using elasto-plastic finite element analysis, thereby exploiting the post-yield reserve strength in structural components, in accordance with the ASME Code, Section III, Division 1, NB-3228 [1] and thereby dispense conventional stress categorization procedures. A practical way of implementing elasto-plastic finite element analysis is by performing limit analysis, where-in the load at which the component undergoes unlimited plastic flow is determined, assuming an elastic-perfectly plastic material behavior, as per NB-3228.1 [1]. An estimate of the load at which a mechanical component would collapse gives an idea of "how safe the operating loads are."

With past industrial design experience being heavily inclined towards conventional stress categorization methods, engineers are treading with caution in implementing the relatively new limit load based design methods. An alternate numerical procedure for determining limit loads based on classical limit analysis theorems would be a welcome step towards applying the elasto-plastic finite element analysis with increased confidence. In this paper, a simple method for determining limit loads of mechanical components, based on a set of linear elastic finite element analyses, is described. Linear elastic analyses based limit load determination had been existing for over a decade [2-4], but application of these methods to industrial problems is yet to be fully exploited. In this paper, limit load of a typical pressure vessel configuration is determined using elasto-plastic finite element analysis and the result is compared with the linear elastic method. The aspects of convergence of linear elastic stress distributions is also studied by following the procedure proposed by Seshadri and Mangalaramanan [5].

2. Determination of Limit Loads using Linear Elastic Finite Element Analysis:

Determination of limit loads using linear elastic finite element analyses is based on the concept of inelastic stress redistribution [3] and the classical lower bound theorem. The procedure involves performing a number of linear elastic finite element analyses. After every analysis, the elastic moduli of all the elements of the structure is modified by using the expression [6]:

$$\left(E_e\right)_i = \left(E_o\right)_i \left[\frac{\sigma_{arb}}{\left(\sigma_{eqv_i}\right)}\right]^q \quad (1)$$

In the above expression, for a given element i :

E_o is the original elastic modulus,
σ_{arb} is some arbitrary stress value and
σ_{eqv} is the von-Mises centroidal equivalent stress of an element and
q is the elastic modulus modification index

Although the repeated elastic analysis procedure is seemingly straight forward, the designer needs to be wary of a number of related problems pertaining to the ability of the method to provide the best lower bound limit load estimate. These problems and the methods of avoiding them have been elaborately discussed by Mangalaramanan [6]. An upper bound multiplier, m^o, given by

$$m^o = \frac{\sigma_y\sqrt{V}}{\sqrt{\sum_{i=1}^{n} \sigma_{ei}^2 \Delta V_i}} \quad (2)$$

is defined [5] as an indicator of the convergence of the stress distributions from linear elastic state to a limit state, where σ_y is the material yield strength and V is the volume of the structure. Should the stress distributions exhibit lack of convergence, a lower value of q in equation (1) would attenuate the extent of elastic moduli modification and thereby enhance convergence. The lower bound limit load of the component is expressed as:

$$P_{LC} = P\left[\frac{\sigma_y}{(\sigma_{eqv})_{max}}\right] \tag{3}$$

where $(\sigma_{eqv})_{max}$ is the maximum von-Mises equivalent stress in the component.

3. Numerical Example

In this section, pressure vessel configurations of practical interest are considered. An isotropic cylindrical pressure vessel with an oblique nozzle is considered for demonstrating the practicability of the reduced modulus method. The method is extended for determining the limit load of orthotropic structures. Although the method proposed is generic, a typical orthotropic circular plate subjected to uniform pressure is considered to validate the proposed methodology. In both the cases the methods could be seen to give good lower bound limit load estimates.

3.1 Cylindrical Pressure Vessel with an Oblique Nozzle:

The procedure mentioned above is applied to a typical pressure component configuration, i.e., a cylindrical pressure vessel with an oblique nozzle as shown in Figure 1. The vessel has an inside radius of 5 inch. and a wall thickness of 2 inch. The thickness of the nozzle is assumed to be 0.5 inch. and the mean radius of the nozzle is calculated by equating the hoop stresses of the nozzle and the vessel. The axis of the nozzle is assumed to be inclined at an angle of 70 degrees with the axis of the vessel. The material of the component is assumed to have a Young's modulus of 30000 ksi, Poisson's ratio of 0.3 and yield strength value of 30000 psi. The problem is modeled using eight noded tetrahedral elements. The finite element mesh is shown in Figure 2. An arbitrary uniform internal pressure of 10 ksi is applied for performing the linear elastic iterations, assuming a q value of 1. The variation of m^oP and P_L with elastic iterations is given in Figure 3. It can be seen that the elastic iterations become unstable after nine iterations. Currently efforts are being directed towards determining before hand the ensuing instability and means to avoid it.

The following are the limit load estimates obtained:

Inelastic finite element analysis: 8000 psi	Run time: 14856 sec.	Elapsed time:117276 sec.
Repeated elastic analysis: 7332 psi	Run time: 8565 sec.	Elapsed time: 33547 sec.

3.2 Orthotropic Circular Plate subjected to Uniform Internal Pressure:

Structural orthotropy becomes an important consideration in engineering, especially in the design of low-weight high strength components. While analyzing tube-sheets in nuclear steam generators, the notion of orthotropy can be used with advantage. The topic of limit analysis of components made out of these materials is therefore important from the view-point of structural design. Conventional design methods such as analytical formulations are restricted to simple configurations. An alternative method for limit load determination can therefore be used for conveniently validating the latest analysis techniques such as inelastic finite element analysis.

For the present study, an axisymmetric circular plate fixed all along the edges, as shown in Figure 4, is considered. Since the problem is one of limit load determination, elastic properties of the component play little role. The material properties that govern the collapse of the structure are the orthotropic yield values. Therefore, an arbitrary uniform Young's modulus of 30000 ksi and a Poisson's ratio of 0.3 are applied. The yield values of the material are assumed to be as follows:

Direction	Yield value (ksi)
$(\sigma_Y)_x$	30
$(\sigma_Y)_y$	25
$(\sigma_Y)_z$	20
$(\tau_Y)_{xy}$	25
$(\tau_Y)_{yz}$	20
$(\tau_Y)_{zx}$	15

In the above table, the subscripts *"Y"* denotes the yield *x*, *y* and *z* denote the directions as indicated in Figure 4. The symbols σ and τ denote the tensile and the shear stresses, respectively.

The problem is modelled using four noded axisymmetric isoparametric elements. An arbitrary load of 100 psi is applied in order to perform the linear elastic iterations. The limit load estimates are shown in Figure 5.

Theoretical Formulation:

In this section, the method for determining the limit loads of orthotropic structures is discussed. The Hill's yield criterion [7] is applied in order to characterize the orthotropy. However, the proposed methodology is generic and could be easily extended to other yield criteria. The Hill's yield criteria is expressed by:

$$f(\sigma_{ij}) = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1 \qquad (4)$$

The parameters *F*, *G*, *H*, *L*, *M* and *N* characterize the current state of orthotropy, and are related to the material yield values as follows:

$$2F = 1/(\sigma_Y)_y^2 + 1/(\sigma_Y)_z^2 - 1/(\sigma_Y)_x^2$$
$$2F = 1/(\sigma_Y)_z^2 + 1/(\sigma_Y)_x^2 - 1/(\sigma_Y)_y^2$$
$$2F = 1/(\sigma_Y)_x^2 + 1/(\sigma_Y)_y^2 - 1/(\sigma_Y)_z^2$$

and $2L = 1/(\tau_Y)_{yz}^2$, $2M = 1/(\tau_Y)_{zx}^2$ and $2N = 1/(\tau_Y)_{xy}^2$.

The elastic modulus modification procedure is maintained the same as for isotropic components, given by equation (1), for simplicity. The function, $f(\sigma_{ij})$ is determined for every element. Classical lower bound theorem is invoked for determining the limit load for every elastic stress distribution as:

$$P_L = \frac{P}{f\left[(\sigma_{ij})_{\max}\right]}$$

where P is the arbitrary applied load for performing the elastic analyses.

4. Conclusion

Limit load determination using repeated elastic analyses form an excellent alternative for verifying the results of inelastic finite element analysis at a fraction of the time and cost. In situations where the component geometry is complex enough to render inelastic analysis impracticable, linear elastic analyses based methods could be used with confidence. It is, however, worth noting from Figure 3 that the variation of classical lower bound results remain no longer monotonic after the ninth iteration, but becomes unstable. The upper bound multiplier, on the other hand, exhibits substantial stability throughout the iteration process. This is a typical Class II component, showing global stability and local instability, as categorized by Seshadri and Mangalaramanan [5]. The authors are currently investigating methods to improve the stability of the elastic iteration procedure. The simplified method has also been extended for analyzing orthotropic structures. To the best of the authors' knowledge, this is the first time the method is being applied for orthotropic components. Given the necessity for limit analysis methods in designing steam generator tubesheets, it is hoped that the proposed method would open the doors for applying reduced modulus methods. Currently, efforts are being directed towards getting improved lower bound limit loads for orthotropic structures, by invoking energy considerations.

References

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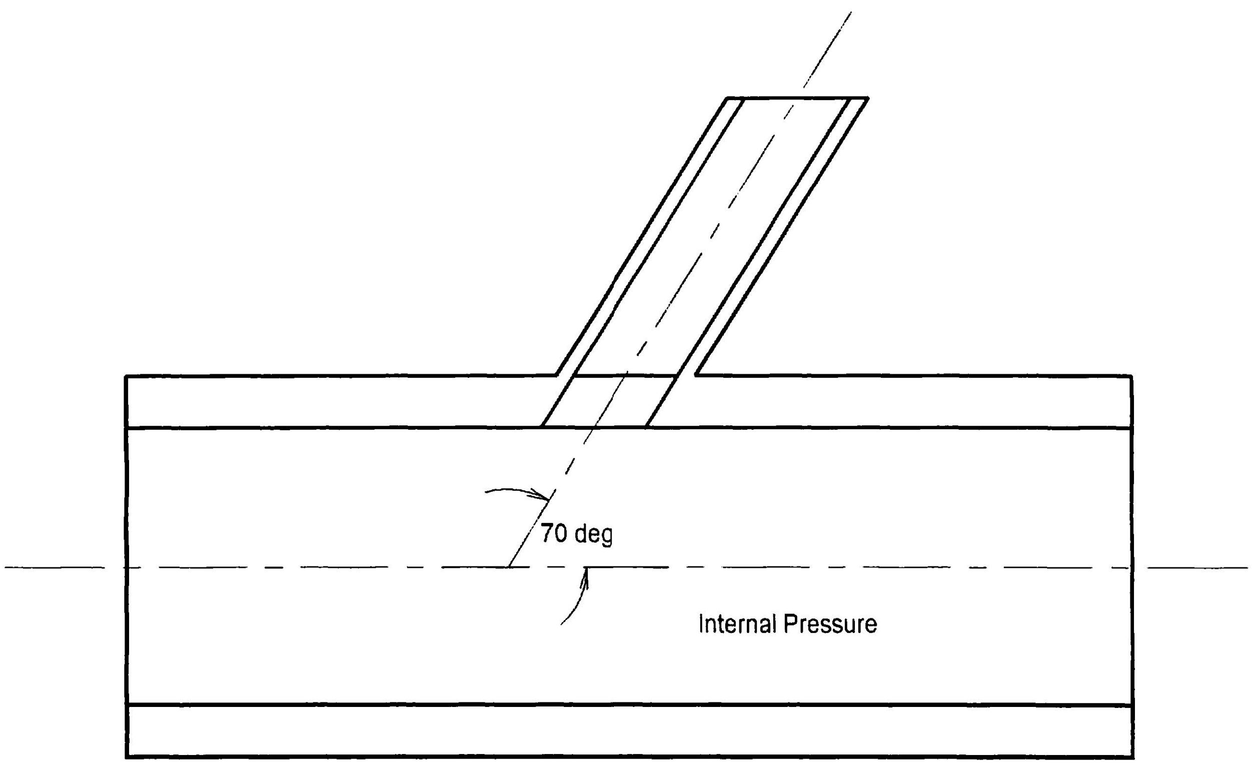

Figure 1: Schematic of a Cylindrical Pressure Vessel with an Oblique Nozzle

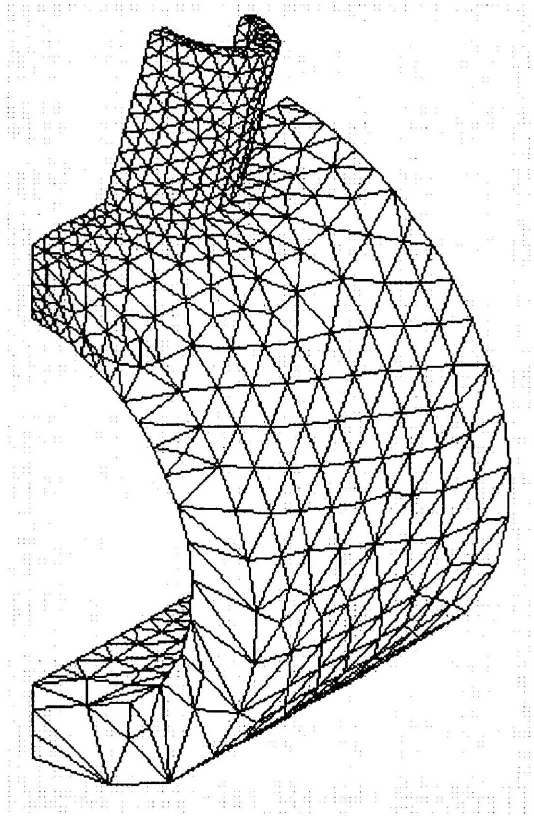

Figure 2: Finite Element Mesh of the Oblique Nozzle/Cylinder Intersection

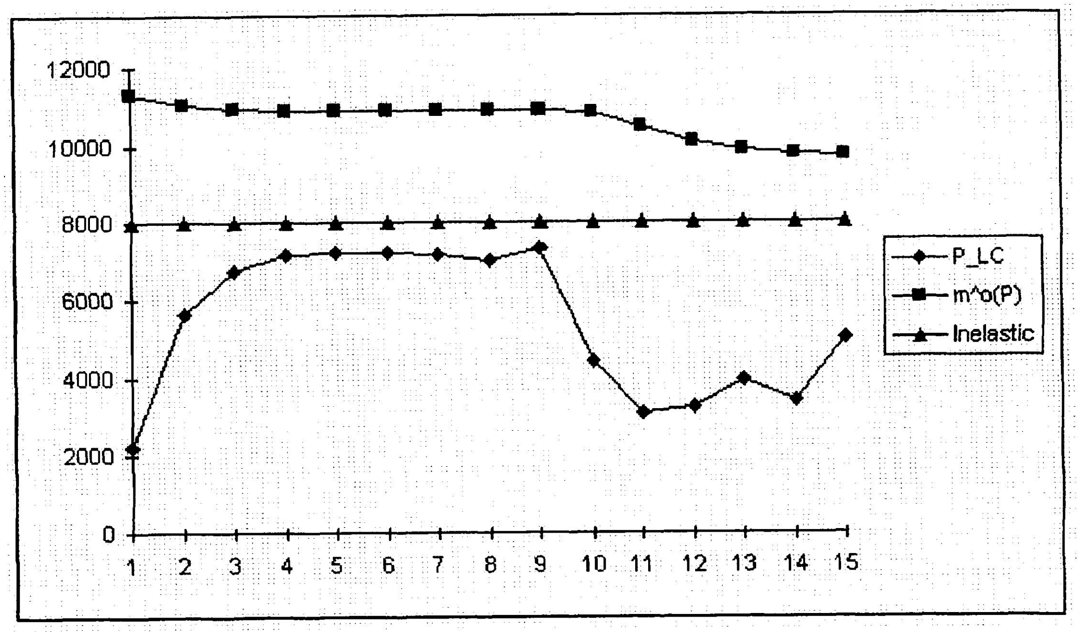

Figure 3: Variation of Limit Loads with Elastic Iterations for the Cylindrical Pressure Vessel with Oblique Nozzle

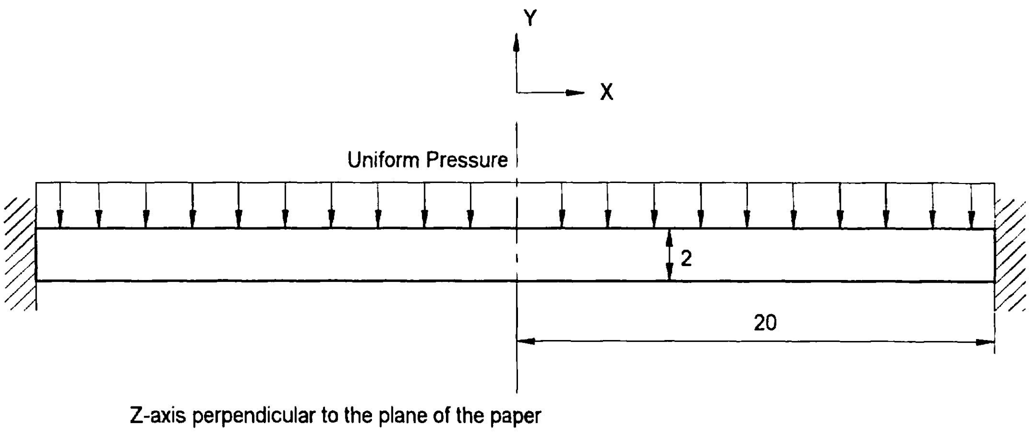

Figure 4: Dimensions of the Orthotropic Fixed Circular Plate subjected to Uniform Pressure

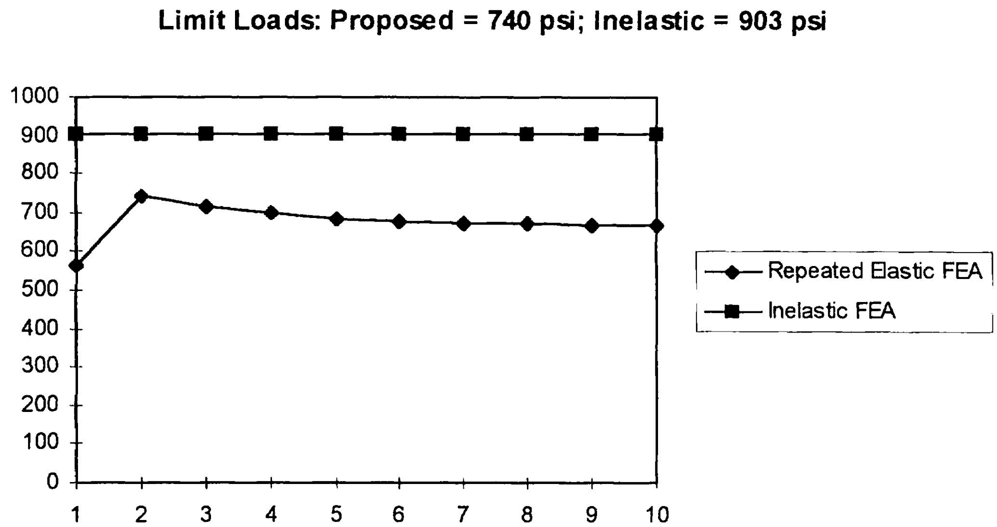

Figure 5: Limit Loads of the Fixed Circular Plate