ENERGY DEPOSITION OF FAST α -PARTICLES IN A FULLY IONIZED DEUTERIUM - TRITIUM PLASMA

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Abstract

The charged-particle-transport equation, in the absence of large-angle scattering and angular dispersion, has been analytically solved for three source distributions in one-dimensional spherical geometry for a homogeneous medium. Energy deposition profiles to ions and electrons for 3.5 Mev particles slowing down in a plasma have been computed for the bench-mark problem of a central isotropic time-independent source distribution. These profiles have been compared with those obtained by using two discrete-ordinates computer codes. Significant differences are found near the peaks and sharp cut-off points of the energy-deposition profiles.

1. INTRODUCTION

The accurate simulation of the transport and energy deposition of fast, charged particles slowing down in a plasma is an important problem for many controlled thermonuclear fusion schemes. For example, the energy deposition of 3.5 MeV α-particles needs to be accurately simulated to investigate the burn-wave propagation in a compressed deuterium-tritium pellet in an inertial confinment laser-fusion scheme. In the inertial confinement schemes, using external ion-beams, the compression and ignition of a pellet depend upon the energy-deposition profiles of the external charged particles. The problem of charged particles slowing down in a hot plasma has been investigated by several workers using various techniques. Corman et al.[1] have derived a flux-limited diffusion equation. Antal and Lee [2] have set up mass and energy balance equations for charged particles and developed a discrete-ordinates (ISN) computer-code to solve the equations in the absence of large-angle scattering for one-dimensional spherical geometry. Haldy and Ligou [3] have solved the Fokker-Planck equation governing the transport of charged particles in the continuous slowing-down approximation by the momentsmethod for a homogeneous medium, in one-dimensional spherical geometry. Mehlhorn and Duderstadt [4] have developed a discrete-ordinates (SN) computer code-TIMEX-FP, to solve the time-dependent Fokker-Planck equation in one-dimensional spherical geometry to obtain solutions for the angular flux and energy deposition. Khandekar and Sahni [5,6] have analytically solved the charged-particle-transport equation for a uniform plasma containing a uniform source of 3.5 MeV α-particles in one-dimensional spherical geometry. A detailed and accurate treatment of the continuous slowing-down and angular dispersion phenomena for charged particles is described by the linear Fokker-Planck equation Haldy and Ligou [7] have developed a multigroup formalism to numerically solve the Fokker-Planck equation by methods similar to those used for neutron transport. A different approach that treats the energy variable in the Fokker-Planck equation more accurately has also been recently developed by Tran and Ligou. As a matter of fact, Mehlhorn and Duderstadt [4] have reported significant differences in the energy-deposition profiles predicted by two different discrete-ordinates computer-codes for a benchmark problem. In this paper, an integral equation has been derived for charged particle transport. This equation is then solved for some benchmark problems in one-dimensional, spherical geometry for isotropic source-distributions and analytical expressions for α-particle number-densities have been obtained. The energy-deposition profiles for one benchmark problem [4] have been obtained by numerical integration over the space variable using an analytical expression for the charged particle number-density. The comparison of these energy-deposition profiles and those obtained from two multigroup discrete-ordinates computer codes reveals that the discrete-ordinates codes make some α-particles deposit their energies at points more than one range away from their source positions. This phenomenon introduced by the multi-group S_N codes is unphysical and is a limitation of this discretization procedure. [8]

Employing the numerical scheme that uses diamond differencing in both space and energy variables, Tran and Ligou[9] have reported more accurate numerical solutions of the Fokker-Planck equation. These solutions do not display the unphysical formation of tails that extend beyond the expected range of fast ions, in the energy deposition curves.

2. CHARGED PARTICLE TRANSPORT EQUATION

In the framework of the continuous slowing-down approximation which models the slowing down and transport of charged particles as a cumulative effect of small angle scattering collisions, the basic equation that governs this phenomenon is the linear Fokker-Planck equation. This equation can be written as:

$$\nabla \cdot \Omega \phi + \frac{1}{v}\frac{\partial \phi}{\partial t} = \frac{\partial (U\phi)}{\partial E} + T\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial \phi}{\partial \mu}\right] + Q \qquad (1)$$

The terms $U(\vec{r}, E, t)$ and $T(\vec{r}, E, t)$ are called the Fokker-Planck coefficients. These coefficients incorporate the detailed physics of continuous interactions between the external charged particles and the ions and electrons constituting a plasma medium. The phenomenon of angular dispersion represented by the term involving T becomes important when the energies of the external charged particles become low and when the mass of the external particles is small compared to that of the plasma particles. The energy-derivative term is a drag term which represents the slowing-down of the external particle, without a change in their directions. When heavy α-particles slow down in a fully ionized stationary hot plasma consisting of lighter particles deuterons, tritons and electrons, the α-particle trajectories can be assumed to be straight lines between two large-angle scattering collisions. Further, in the continuous slowing-down approximation [10] the effect of Coulomb collisions can be accounted for by an acceleration $\vec{a}(\vec{r}, E, t)$ [11]. The phenomenon of angular dispersion can be neglected in this case, and the transport of charged particles is governed by a simplified equation.

The α-particle transport equation can then be written as

$$\frac{\partial N}{\partial t} + \nabla_r \cdot (\vec{v} N) + \nabla_v \cdot (\vec{a} N) = S - \sigma_t (v N) \qquad (2)$$

Here, $N(\vec{r}, \vec{v}, t)$ is the number-density of the α-particles in phase-space; $\vec{r}$ denotes the space coordinates and $\vec{v}$ denotes the velocity with respect to a stationary reference frame; σ_t is the macroscopic total cross-section for large-angle collisions; $S(\vec{r}, \vec{v}, t)$ is the total source strength of α-particles. Eq. (2) can be written as:

$$\frac{\partial N}{\partial t} + \vec{v} \cdot (\nabla_r N) + \vec{a} \cdot (\nabla_v N) + \left(\sigma_t + \frac{\nabla_v \cdot \vec{a}}{v}\right) N v = S \qquad (3)$$

Now the variables, $\vec{r}$, $\vec{v}$ and t can be changed to the initial coordinates $\vec{r}_o$, $\vec{v}_o$ and t_o of α-particles. These changed variables are related to the old ones by:

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = \vec{a} \qquad (4)$$

with the initial conditions;

$$\vec{v}_o = \vec{v}(t_o) \qquad (5)$$

$$\vec{r}_o = \vec{r}(t_o) \qquad (6)$$

Let $N[\vec{r}(\vec{r}_o,\vec{v}_o,t),\vec{v}(\vec{r}_o,\vec{v}_o,t),t] = M(\vec{r}_o,\vec{v}_o,t)$ (7)

and let a cross-section σ be defined by:

$$v(\vec{r}_o,\vec{v}_o,t)\,\sigma(\vec{r}_o,\vec{v}_o,t) = v\left[\sigma_t(\vec{r},\vec{v},t) + \frac{\nabla_v\cdot\vec{a}}{v}\right] \quad (8)$$

Then, from Eq.(2) we get:

$$\frac{dM}{dt} = S - \sigma v M \quad (9)$$

This equation can be integrated from t_o to t to obtain:

$$M(\vec{r}_o,\vec{v}_o,t)\exp\left[\int^{t}\sigma(\vec{r}_o,\vec{v}_o,t')\,v\,dt'\right] - M(\vec{r}_o,\vec{v}_o,t_o)\exp\left[\int^{t_o}\sigma(\vec{r}_o,\vec{v}_o,t')\,v\,dt'\right]$$
$$= \int_{t_o}^{t} S\left[\vec{r}'(\vec{r}_o,\vec{v}_o,t'),\vec{v}'(\vec{r}_o,\vec{v}_o,t'),t'\right] \times \exp\left[+\int^{t'} v(\vec{r}_o,\vec{v}_o,t'')\,\sigma(\vec{r}_o,\vec{v}_o,t'')\,dt''\right]dt' \quad (10)$$

The α-particle number-density can then be obtained as:

$$N(\vec{r},\vec{v},t) = N(\vec{r}_o,\vec{v}_o,t_o)\exp\left[-\int_{t_o}^{t} v(\vec{r}_o,\vec{v}_o,t')\,\sigma(\vec{r}_o,\vec{v}_o,t')\,dt'\right]$$
$$+ \int_{t_o}^{t} S\left[\vec{r}'(\vec{r}_o,\vec{v}_o,t'),\ \vec{v}'(\vec{r}_o,\vec{v}_o,t'),\ t'\right] \times \exp\left[-\int_{t'}^{t} v''(\vec{r}_o,\vec{v}_o,t'')\,\sigma(\vec{r}_o,\vec{v}_o,t'')\right]dt' \quad (11)$$

For better physical insight, the time variable, t', can be changed to $\tau' = t - t'$. The velocity and position as a function of the new time variable then are given by:

$$\frac{d\vec{r}'(\tau')}{d\tau'} = -\vec{v}'(\tau') \quad (12)$$

$$\frac{d\vec{v}'(\tau')}{d\tau'} = -\vec{a}\left[\vec{r}'(\tau'),\vec{v}'(\tau'),\tau'\right] \quad (13)$$

with the initial conditions:

$$\vec{r}'(\tau'=t-t_0) = \vec{r}(t_0) \tag{14}$$

$$\vec{v}'(\tau'=t-t_0) = \vec{v}(t_0) \tag{15}$$

The α-particle number-density is then governed by:

$$\begin{aligned} N(\vec{r},\vec{v},t) &= N\left[\vec{r}_0(\vec{r},\vec{v},t),\vec{v}_0(\vec{r},\vec{v},t),t_0\right] \\ &\quad \times \exp\left[-\int_0^{t-t_0} v''(\tau'')\sigma\left\{\vec{r}''(\vec{r},\vec{v},\tau''),\vec{v}''(\vec{r},\vec{v},\tau''),t-\tau''\right\}d\tau''\right] \\ &\quad + \int_0^{t-t_0} S\left[\vec{r}'(\vec{r},\vec{v},\tau'),\vec{v}'(\vec{r},\vec{v},\tau'),t-\tau'\right] \\ &\quad \times \exp\left[-\int_0^{\tau'} v''(\tau'')\sigma\left\{\vec{r}''(\vec{r},\vec{v},\tau''),\vec{v}''(\vec{r},\vec{v},\tau''),t-\tau''\right\}d\tau''\right]d\tau' \end{aligned} \tag{16}$$

The initial time t_0 can be chosen such that:

$$N(\vec{r}_0,\vec{v}_0,t_0) = 0 \tag{17}$$

The source is assumed to be introduced in the system at the instant $t=0$ such that

$$S(\vec{r},\vec{v},t)=0, \text{ for } t<0, \text{ for all } \vec{r} \text{ and } \vec{v}. \tag{18}$$

The α-particle transport equation can then be written in a physically more transparent integral form:

$$N(\vec{r},\vec{v},t)=\int_0^t S\left[\vec{r}'(\vec{r},\vec{v},\tau'),\vec{v}'(\vec{r},\vec{v},\tau'),t-\tau'\right]\times\exp\left[-\int_0^{\tau'} v''(\tau'') \times\sigma\left\{\vec{r}''(\vec{r},\vec{v},\tau''),\vec{v}''(\vec{r},\vec{v},\tau''),t-\tau''\right\}d\tau''\right]d\tau' \tag{19}$$

The number-density, $N(\vec{r},\vec{v},t)$ is thus made up of contributions from particles that start at earlier times $t-\tau'$ with the appropriate position $\vec{r}'(\vec{r},\vec{v},\tau')$ and velocity $v'(\vec{r},v,\tau')$ and which do not suffer a "collision" (with the cross-section σ as defined in Eq.(8)) on their way to the current position $\vec{r}$ and velocity $\vec{v}$ at the current time t . The distance travelled, the time spent and the corresponding change in velocity are correlated through relations (12) and (13).

Analytical solutions to Eq.(19) for a general problem of space and time-dependent sources, in spatially inhomogeneous media, in a three-dimensional geometry, in the presence of large-angle scattering would be very difficult to obtain. Certain benchmark problems have been solved in this paper under simplifying assumptions. The D-T plasma has been assumed to be fully ionized and uniform; the sources of α-particles are assumed to be isotropic and monoenergetic. Large-angle scattering has been neglected. Then [2]

$$\vec{a}(\vec{r},\vec{v},t) = -a(v)\frac{\vec{v}}{v}$$

$$\vec{a}(\vec{r},\vec{v},t) = -a(v)\vec{\Omega} \quad \text{where } a(v) \geq 0$$

$$\sigma_t(\vec{r},\vec{v},t) = 0$$

Under these conditions, the α-particle trajectories will be straight lines and the scalar distance travelled and the time elapsed will be determined solely by the initial and final speeds. As an α-particle slows down from a speed v' to v, the scalar distance travelled by it is given by:

$$s(v',v) = \int_v^{v'} \frac{v''}{a(v'')}\,dv'' \tag{20}$$

Let $s_0(\vec{r},\vec{\Omega})$ be the distance from the point $\vec{r}$ to the boundary of a convex region along rhe direction $-\vec{\Omega}$. If $s(v',v)$ turns out to be greater than $s_0(\vec{r},\vec{\Omega})$, it implies that α-particles can not slow down from a speed v' to the speed v within this convex region when they reach the position $\vec{r}$. The time interval elapsed in slowing down from the speed v' to the speed v is given by:

$$\tau = \int_v^{v'} \frac{dv''}{a(v'')} \tag{21}$$

Let us define:

$$N(\vec{r},\vec{v},t)\,d\vec{v} = N(\vec{r},v,\vec{\Omega},t)\,dv\,d\vec{\Omega}$$

$$N(\vec{r},\vec{v},t) = \frac{1}{v^2}N(\vec{r},v,\vec{\Omega},t)$$

$$S(\vec{r},\vec{v},t)\,d\vec{v} = S(\vec{r},v,\vec{\Omega},t)\,dv\,d\vec{\Omega}$$

$$S(\vec{r},\vec{v},t) = \frac{1}{v^2}S(\vec{r},v,\vec{\Omega},t) \tag{22}$$

Then, changing the variable of integration in (19) from τ' to v' and assuming no incoming current of α-particles at the boundary of the convex region, the number-density obeys the equation:

$$\frac{1}{v^2} N(\vec{r}, v, \vec{\Omega}, t)$$
$$= \int_v^\infty \left(\frac{1}{v'^2}\right) S\left[\vec{r} - s(v', v)\vec{\Omega}, v', \vec{\Omega}, t - \tau'(v')\right]$$
$$\times \exp\left[-\int_v^{v'} v''\sigma\left\{\vec{r}''(\tau''), v'', t - \tau''(v'')\right\} \frac{dv''}{a(v'')}\right]$$
$$\times \theta\left[s_0(\vec{r}, \vec{\Omega}) - s(v', v)\right] \frac{dv'}{a(v')} \qquad (23)$$

Here $\theta(x)$ is a step function defined as: $\theta(x) = 0$ when $x < 0$, and $\theta(x) = 1$ when $x \geq 0$.
Khandekar and Sahni [5,6] have obtained a solution to Eq.(2) for a uniform, isotropic source, in a homogeneous plasma in the absence of large-angle scattering; in one-dimensional spherical geometry, by using certain substitutions. Analytical expressions can be obtained for certain other source distributions as well, as shown in the next section.

For a homogeneous isotropic plasma, neglecting large-angle scattering

$$\sigma(v) = \frac{\nabla_v \cdot \vec{a}(v)}{v} = \frac{-2a(v)}{v^2} - \frac{1}{v}\frac{da(v)}{dv} \qquad (24)$$

$$\int_v^{v'} \frac{-\sigma(v'')v''}{a(v'')} dv'' = \int_v^{v'} \left[\frac{2}{v''} + \frac{1}{a(v'')}\frac{da(v'')}{dv''}\right] dv''$$
$$= \ln\left[\frac{v'^2}{v^2}\frac{a(v')}{a(v)}\right] \qquad (25)$$

The α-particle number-density is then given by:

$$N(\vec{r}, v, \vec{\Omega}, t)$$
$$= \frac{1}{a(v)} \int_v^\infty S\left[\vec{r} - s(v', v)\vec{\Omega}, v', \vec{\Omega}, t - \tau'\right]$$
$$\times \theta\left[s_0(\vec{r}, \vec{\Omega}), - s(v', v)\right] dv' \qquad (26)$$

3. SOLUTION TO BENCHMARK PROBLEMS

Problem I: Central isotropic, monoenergetic source in a uniform plasma in one-dimensional spherical geometry

For a uniform isotropic, time-independent source of 3.5 MeV α-particles, localized to a spherical region of radius R_0 , at the centre of a uniform spherical plasma of Radius R; the source strength is given by

$$S(\vec{r}, v, \vec{\Omega}) = \delta(v_0 - v)\,\theta(R_0 - |\vec{r}|) \tag{27}$$

Integrating Eq.(26) over $\vec{\Omega}$ for this source distribution, the steady-state number-density for α-particle for this problem is given by:

$$N(\vec{r}, v) = \frac{1}{a(v)} \int_v^{\infty} \int_{4\pi} \delta(v_0 - v')\,\theta\left[R_0 - |\vec{r} - s(v', v)\vec{\Omega}|\right] \times \theta\left[s_0(\vec{r}, \vec{\Omega}) - s(v', v)\right] d\vec{\Omega}\, dv' \tag{28}$$

For the sphere with radius R_0 the step-function $\theta[s_0(\vec{r}, \vec{\Omega}) - s(v', v)]$ can be conveniently expressed by the equation:

$$\theta\left[s_0(\vec{r}, \vec{\Omega}) - s(v', v)\right] = \theta\left[R - |\vec{r} - s(v', v)\vec{\Omega}|\right]$$

Then

$$N(\vec{r}, v) = \frac{1}{a(v)} \int_v^{\infty} \int_{4\pi} \theta\left[R_0 - |\vec{r} - s(v', v)\vec{\Omega}|\right] \theta\left[R - |\vec{r} - s(v', v)\vec{\Omega}|\right] \times \delta(v_0 - v)\, d\vec{\Omega}\, dv' \tag{29}$$

Taking $\vec{r}$ as the polar axis, the integration over v' and the solid-angle can be carried out to obtain:

$$N(r, v) = \frac{4\pi}{a(v)} \quad \text{when } r < R_0 \text{ and } s(v_0, v) < R_0 - r$$

$$N(r, v) = \frac{\pi}{a(v)} \frac{R_0^2 - [r - s(v_0, v)]^2}{r\, s(v_0, v)}$$

$$\text{when } r < R_0 \text{ and } R_0 - r < s(v_0, v) < R_0 + r$$

$$N(r, v) = 0, \quad \text{when } r < R_0 \text{ and } s(v_0, v) > R_0 + r \tag{30}$$

Outside the source region ($r > R_0$) the α-particle number-density is given by:

$$N(r,v) = \frac{\pi}{a(v)} \; \frac{R_0^2 - [r - s(v_0,v)]^2}{r s(v_0,v)}$$

when $r - R_0 < s(v_0,v) < r + R_0$,

$$N(r,v) = 0 \quad \text{otherwise.} \qquad (31)$$

These analytical expressions have been used in sect.4 to obtain the energy deposition profiles for this problem.

Problem II: Concentric shell source in a uniform spherical plasma

For an isotropic, time-independent source of 3.5 MeV α-particles confined to a spherical shell of inner and outer radii R_1 and R_2 respectively, present in a uniform plasma of radius $R(R > R_2 > R_1)$, the source strength is given by:

$$S(\vec{r},v,\vec{\Omega}) = \delta(v_0 - v)\,\theta(r - R_1)\,\theta(R_2 - r) \qquad (32)$$

The analytical expression for the α-particle number-density, in this case, turns out to be:

$$\begin{aligned} N(r,v) = \frac{2\pi}{a(v)} \left\{ \frac{1}{2 r s(v_0,v)} \right\} \Big[& \left\{ R_2^2 - (r - s(v_0,v))^2 \right\} \\ & \times \theta\left\{ (r - s(v_0,v))^2 - R_1^2 \right\} \theta\left\{ R_2^2 - (r - s(v_0,v))^2 \right\} \\ & - \left\{ R_2^2 - (r + s(v_0,v))^2 \right\} \theta\left\{ (r + s(v_0,v))^2 - R_1^2 \right\} \\ & \times \theta\left\{ R_2^2 - (r + s(v_0,v))^2 \right\} + \left\{ R_2^2 - R_1^2 \right\} \\ & \times \theta\left\{ (r + s(v_0,v))^2 - R_1^2 \right\} \theta\left\{ R_1^2 - (r - s(v_0,v))^2 \right\} \Big] \end{aligned} \qquad (33)$$

Problem III: A boundary source incident uniformly and radially inwards at the boundary of a uniform spherical plasma

The source density for a monoenergetic, surface-source incident uniformly and radially inwards on the surface of a spherical plasma of radius R is given by:

$$S(\vec{r},v,\vec{\Omega}) = \frac{1}{2\pi}\,\delta(v_0 - v)\,\delta(r - R)\,\delta(\vec{r}\cdot\vec{\Omega} + 1) \qquad (34)$$

Eq.(26) can be integrated over $\vec{\Omega}$ to obtain the α-particle number-density expression:

$$N(r,v) = \frac{R}{a(v)\, r s(v_0,v)}\, \delta\left\{ \frac{r^2 - (R - s(v_0,v))^2}{2Rs(v_0,v)} \right\} \quad (35)$$

4. ENERGY-DEPOSITION PROFILES FOR A BENCHMARK PROBLEM

The work done by the deceleration force due to electrons of the plasma acting on α-particles lying in the range, $\vec{r}$ to $\vec{r} + d\vec{r}$, $\vec{v}$ to $\vec{v} + d\vec{v}$ is given by:

$$dW = (m_\alpha \vec{a}_e \cdot \vec{v})\, N(\vec{r}, v)\, dv\, d\vec{r} \quad (36)$$

The stopping power U = dE/dr per unit ion or electron number-density is related to the deceleration by:

$$a(v) = \frac{U}{m_\alpha}$$

Let N_e and N_i be the number-densities of electrons and ions in the plasma respectively, and let the energies E_1 and E_2 be defined by:

$$R_0 - r = \int_{E_1}^{E_0} \frac{dE}{N_e U_e + \sum_i N_i U_i} \quad (37)$$

$$R_0 + r = \int_{E_2}^{E_0} \frac{dE}{N_e U_e + \sum_i N_i U_i} \quad (38)$$

Using the analytical expressions for the α-particle number-density, the energy deposited to ions in a spherical shell between $r = r_1$ and $r = r_2$ $(r_2 > r_1)$ in problem I can be shown to be:

$$E_i(r_1, r_2) = \int_{r_1}^{r_2} 4\pi r^2 dr \int_{E_2}^{E_1} \frac{\pi\left(\sum_i N_i U_i(E)\right)}{N_e U_e + \sum_i N_i U_i} \left[\frac{R_0^2 - (r - s(E_0,E))^2}{r s(E_0,E} \right] dE$$

$$+ \int_{r_1}^{r_2} 4\pi r^2 dr \int_{E_1}^{E_0} \frac{4\pi \sum_i N_i U_i(E)\, dE}{N_e U_e + \sum_i N_i U_i} \quad (39)$$

A Similar expression holds for the energy-deposition to electrons. Published results for problem I assumed a source region of radius R_0 = 0.7742 cm;

the density for the D-T (50% - 50%) plasma was $\rho = 0.2125$ g/cm^3; the electron and ion temperatures $\theta e = \theta i = 50$ KeV, the α-particle energy $E_o = 3.5$ MeV. In our case, the integrals in Eq.(39) were evaluated numerically by the trapizoidal rule using the above parameters. The expressions given in Ref.[2] were used for the ion and electron stopping-powers. The energy deposited to ions and electrons was determined in equal radial zones of thickness 0.7742 cm. Energy-deposition profiles have been worked out for this benchmark problem by many authors using numerical methods for solving the charged particle transport equation.

Fig. 1 and 2 compare the energy-deposition profiles for problem I, for slowing down by ions and electrons respectively, using our results and those obtained by using Cooper and Evan's[12] code as reported by Antal and Lee[13] . Fig. 3 and 4 compare our results with the energy-deposition profiles reported by Mehlhorn and Duderstadt obtained using TIMEX-FP (straight line). The Coulomb Logarithms used for results in Figs. 3 and 4 were $ln\Lambda e = 8.25$ and $ln\Lambda i = 22.687$ to match with those of Ref [4].

On comparing the results of ISN computations from Fig. 10 of Ref.[4] and Fig. 4 of Ref.[13] , both ISN results appear to be identical. The differences reported in [4] between TIMEX-FP (straight line) and the ISN results, therefore, seem to be caused by the differences in the values of the Coulomb logarithms used in the two computations.

DISCUSSION AND CONCLUSION

It can be clearly seen from Figs. 1 and 2 that the analytical results and those from Cooper and Evans' code [12, 13] agree quite well except for the zone near the sharp cut-off in the energy-deposition to ions. At such field points (nearly one range away from the source region), most of the charged-particle number-density is contributed by the α-particles moving in a narrow cone of directions centered around $\mu = 1$. Thus, we conjecture that this disagreement would disappear if a larger number of directions were included around $\mu = 1$ in the numerical integration scheme of Cooper and Evans [12]. It has been suggested that the multigroup discrete-ordinates method is not well suited to the solution of problems involving localized sources [14] . Figs. 3 and 4 show the extent to which the S_N results may be off from the accurate results. Physically, an α-particle can not deposit its energy at a point more than one range away from its point of origin. This sharp cut-off, and also the peak energy deposition to ions, is poorly reproduced by the two multigroup discrete-ordinates codes. The S_N codes also fail to accurately reproduce the sharp rise in the energy-deposition to electrons near the position of the Bragg peak.

The inability of the S_N method to accurately solve α-particle transport problems is not surprising. These methods are primarily designed for solving neutron transport problems where large-angle scattering is predominant and angular fluxes nearly isotropic. As a matter of fact, the so-called curvature coefficients in spherical geometry have been explicitly obtained to yield the correct solutions for isotropic angular fluxes. For the isolated source α-particle transport problem treated above, the angular flux is highly anisotropic especially near the sharp cut-off one range away from the source. We feel this is an important reason for the disagreement between the S_N solutions and the analytical results. Tran and Ligou [9] attribute this type of numerical diffusion to the first order approximation applied to the energy variable in the multi-group treatment. If a more accurate diamond scheme is used to treat the space and energy variables, the energy deposition profiles do not display a tail beyond the expected range of fast ions.

REFERENCES

[1] Corman, E.G., W.E. Loewe, G.E. Cooper, A.M. Winslow: Nucl. Fusion 15, (1975) 377

[2] Antal, M.J., C.E. Lee: J. Comput.Phys. 20 (1976) 298

[3] Haldy P.A., J.Ligou: Nucl. Fusion 17 (1977) 1225

[4] Mehlhorn T.A., J.J. Duderstadt: J.Comput.Phys. 38 (1980) 86

[5] Khandekar D.C., D.C.Sahni: Phys.Fluids 23 (1980) 1464

[6] Khandekar D.C., D.C.Sahni: Phys.Lett. 78A (1980) 259

[7] Haldy, P.A., J.Ligou: Nucl. Sci.Eng. 74, (1980) 178

[8] Paranjape, S.D., Khandekar, D.C., D.C.Sahni: ATOMKERNENERGIE/KERNTECHNIK, 41 (1982) 18

[9] Tran, T.M., J. Ligou : Nucl. Sci.Eng. 79 (1981) 269

[10] Ligou, J.: In "Fusion Reaction Product Transport in Inertially Confined Plasmas" (Proc. Int.Topical Meeting on Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems, Munich, 1981) Vol. 2, p. 385

[11] Evans, F.: Phys.Fluids, 16, (1973) 1011

[12] Cooper, R.S., F.Evans: Phys.Fluids, 18 (1975) 332

[13] Antal, M.J., C.E. Lee: Nucl.Sci.Eng. 64 (1977) 379

[14] Ligou, B.J., P.A. Haldy : ATOMKERNENERGIE 32 (1978) 90.

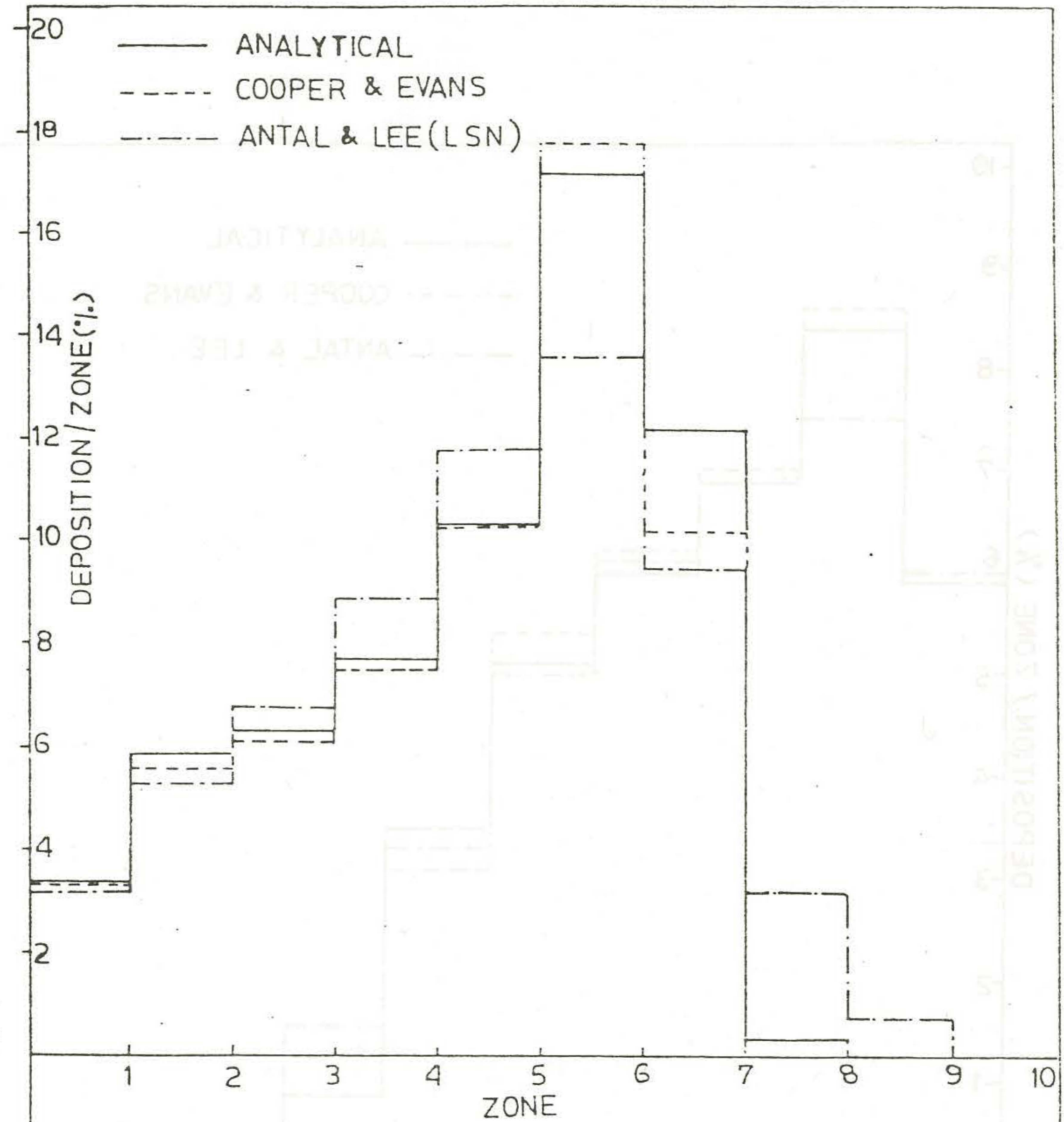

FIG. 1 : PERCENTAGE ENERGY DEPOSITION TO IONS FOR PROBLEM I.

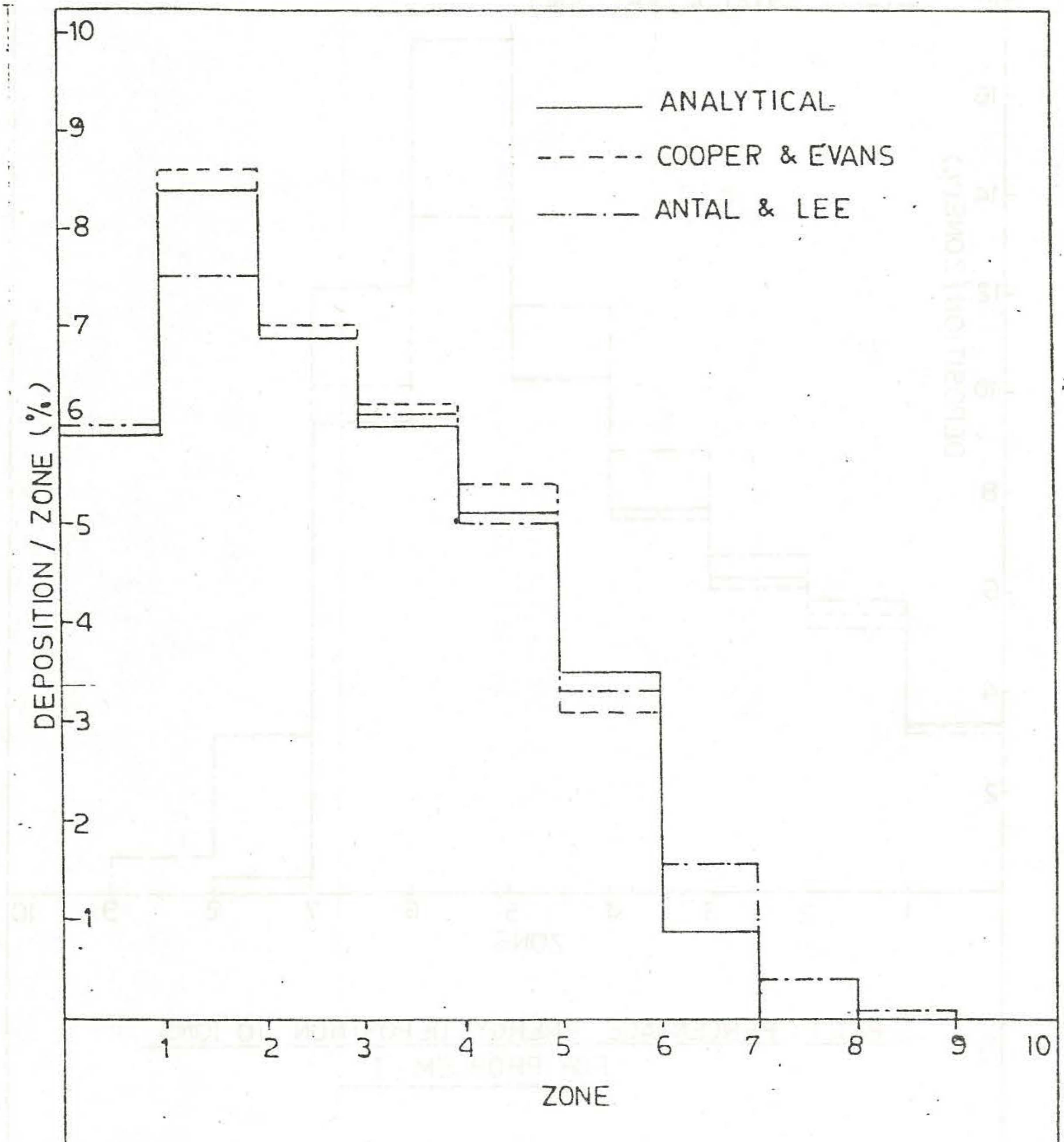

FIG. 2: PERCENTAGE ENERGY-DEPOSITION TO ELECTRONS FOR PROBLEM I

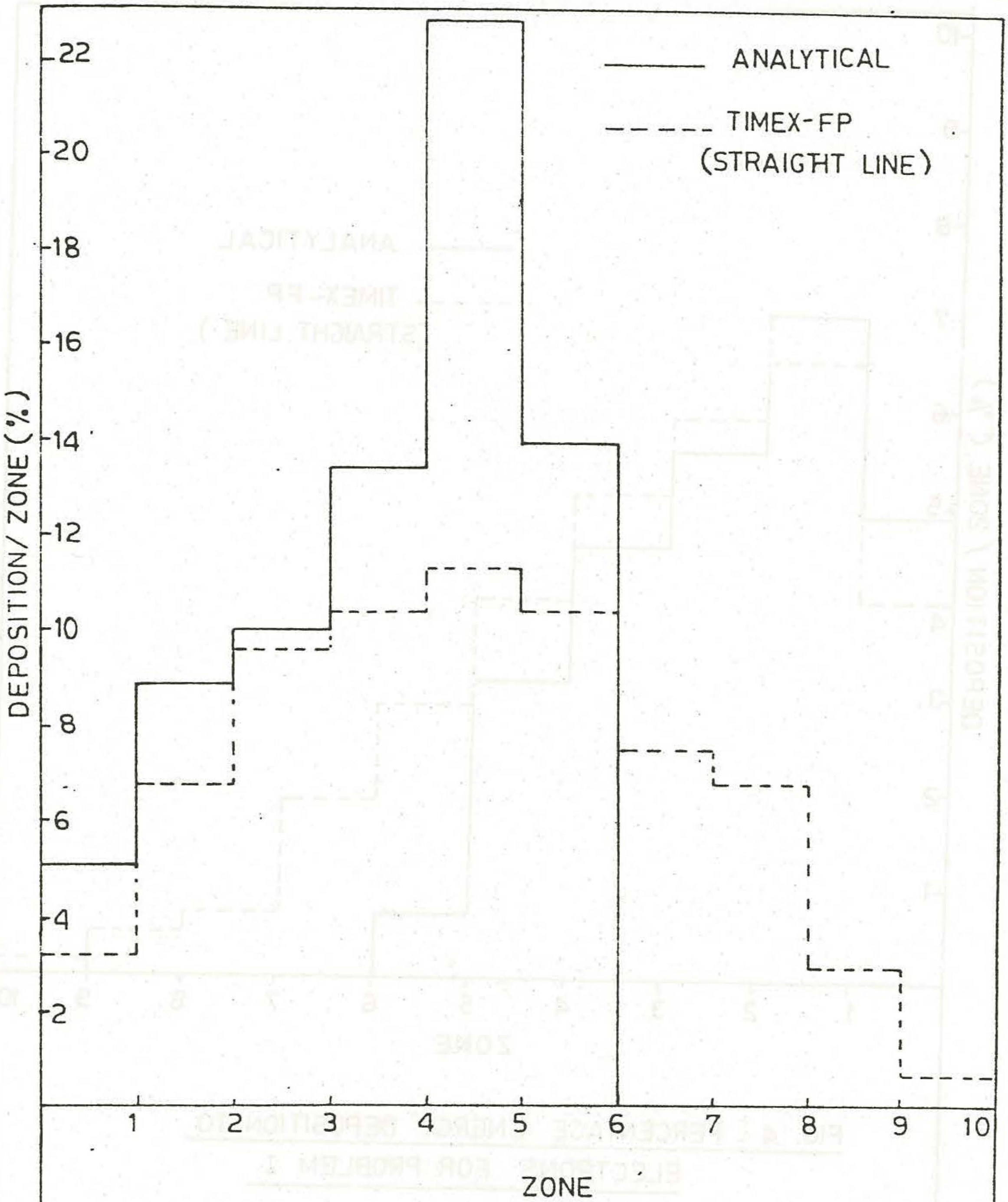

FIG.:3 PERCENTAGE ENERGY DEPOSITION IN IONS FOR PROBLEM I

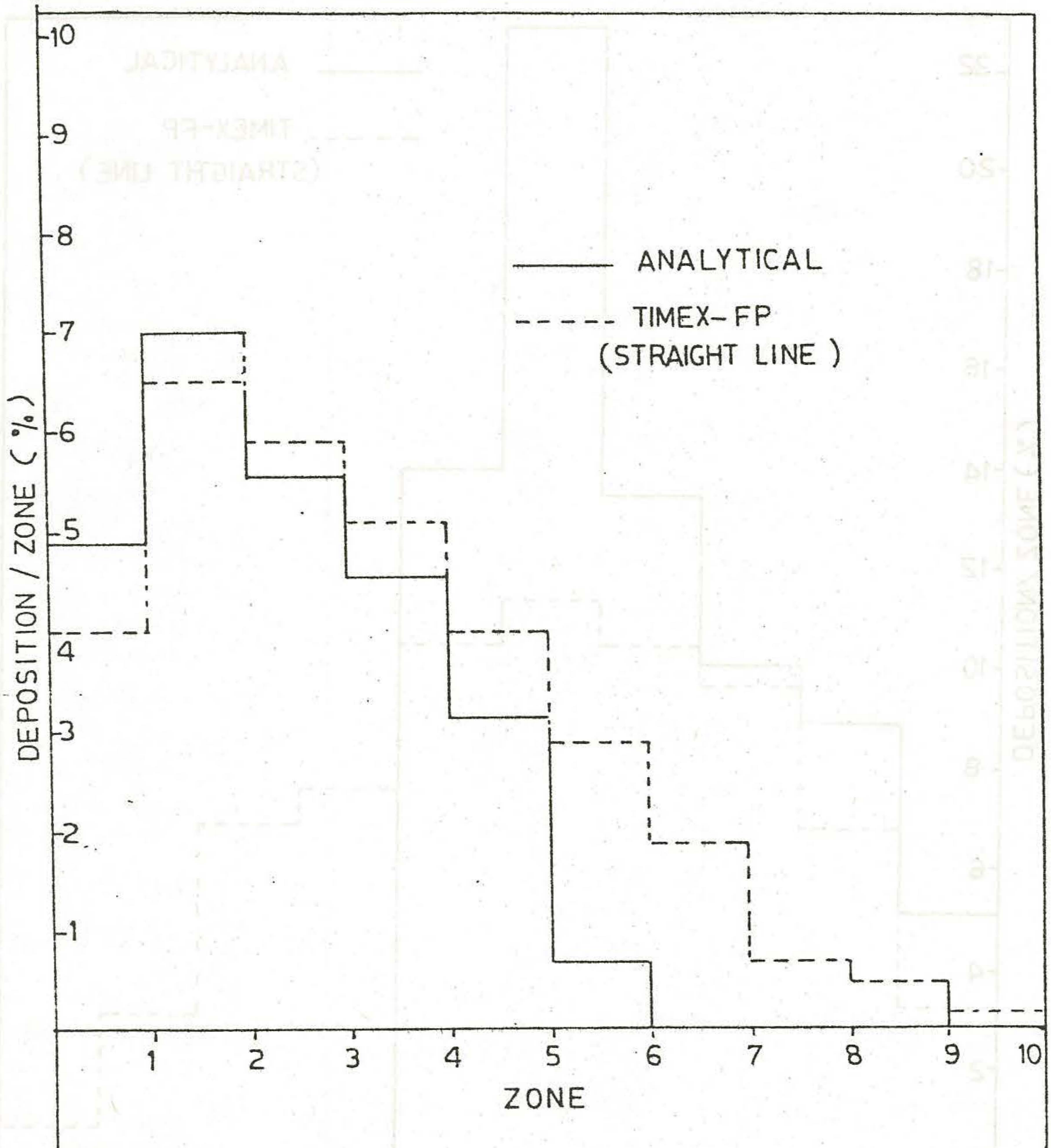

FIG. 4 - PERCENTAGE ENERGY DEPOSITION TO ELECTRONS FOR PROBLEM I