A MULTI-POINT KINETICS MODEL APPLIED TO THREE-DIMENSIONAL NEUTRONIC TRANSIENT ANALYSIS

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ABSTRACT

A computer code for solving the three-dimensional reactor neutronic transient problems by a coupled reactor kinetics method recently derived has been developed and for evaluationg its applicability applied to 3-D kinetic benchmark problems. The performance of the method and code has been compared with the results by the computer codes employing the direct fine and coarse mesh methods.

1. INTRODUCTION

The analysis of power distribution under normal transient operation as well as accident conditions in large power reactors often requires the use of detailed space-time neutron diffusion calculations. Especially in CANDU reactor, the information of zonal power behaviors during normal operation is important for reactor reguration. Efforts have been directed toward the use and development of multi-dimensional few group transient diffusion programs. However, in large power reactors, full 3-D analysis by directly solving the neutron diffusion equation requires large computing time even with modern high speed computer. Therefore, simple methods requiring small computing time have been tried and developed. The coupled reactor kinetics method chosen in this work belongs to such category.

Principle objective of this work is to make a 3-D neutronic model for simulator which fast computing time as possible is desirable and aimed at developing a model providing reliable solutions to relevant problems. The multi-point kinetics method[1] of the coupled reactor theory recently derived has been chosen and a program developed, and for evaluating its efficiency in simulator neutronic-model tested with the typical 3-D kinetics benchmark problems[2-5].

Model equations are exact kinetic equations for fission sources in the multi-coupled regions or reactors with six delayed neutron precursor groups. The benchmark problems chosen are typical LWR and CANDU reactor transient problems with asymmetric reactivity insertions. In Section 2, the model equations are briefly described, in Section 3, the computational procedure is described, and in Section 4 and 5, the test results and some conclusions are given.

2. MODEL DESCRIPTION

The model in the reference[1] is briefly described. The derivation of the coupled reactor kinetic method begins with the time-dependent multi-group diffusion equations for perturbed system with delayed neutrons,

$$\frac{1}{v_g}\frac{\partial \phi_g(r,t)}{\partial t} = \left(-A' + \frac{(1-\beta)}{k_0}B'\right)\phi_g(r,t) + \sum_i \chi_{ig}^{d}\lambda_i C_i(r,t) \qquad (1)$$

and

$$\frac{\partial C_i(r,t)}{\partial t} = \frac{1}{k_0}\beta_i F'\phi_g(r,t) - \lambda_i C_i(r,t). \tag{2}$$

Here A' and B^p are the removal and prompt fission operators involving the perturbation, δA and δF, respectively, and given by

$$\begin{aligned} A'\phi_g(r,t) &= (A+\delta A)\phi_g(r,t) \\ &= \{-\nabla\cdot D_g(r,t)\nabla + \Sigma_{rg}(r,t)\}\ \phi_g(r,t) - \sum_{g'}\Sigma_{sg'\to g}(r,t)\phi_{g'}(r,t) \end{aligned} \tag{3}$$

and

$$B^p\phi_g(r,t) = \chi_g^p F'\phi_g(r,t) = \chi_g^p(F+\delta F)\phi_g(r,t) = \chi_g^p\sum_{g'}\nu\Sigma_{fg'}(r,t)\phi_{g'}(r,t). \tag{4}$$

In the coupled reactor kinetics method[1], inhomogeneous importance functions for unperturbed system as given by Eq. (5) below are used.

$$\begin{aligned} A^+G_{mg}(r) &= \{-\nabla\cdot D_g(r,0)\nabla + \Sigma_{rg}(r,0)\}\ G_{mg}(r) - \sum_{g'}\Sigma_{sg'\to g}(r,0)G_{mg'}(r) \\ &= \nu\Sigma_{fg}(r,0)\delta_m(r), \end{aligned} \tag{5}$$

where $G_{mg}(r)$ denotes the importance function for group g and a region (or reactor) m of N regions (or reactors) system, and $\delta_m(r)$ is defined such that $\delta_m(r) = 1$ when $r \in V_m$ and $\delta_m(r) = 0$, elsewhere.

Applying the adjoint operator to the time-dependent multi-group diffusion equation, (1) with the functions introduced, $G_{mg}(r)$, we obtain the multi-point kinetics equations for the fission densities $S_m(t)$ with some parameters defined as follows:

$$\begin{aligned} l_m(t)\frac{dS_m(t)}{dt} &= -\{1-\Delta k_m^F(t)\}S_m(t) + \sum_{n=1}^{N}\sum_i k_{imn}^d(t)\lambda_i C_{in}(t) \\ &\quad + \sum_{n=1}^{N}[\frac{1}{k_0}\{1-\beta_{mn}(t)\}k_{mn}^p(t) - \Delta k_{mn}^A(t)]S_n(t). \end{aligned} \tag{6}$$

Here, $k^p_{mn}(t)$ and $k^d_{imn}(t)$ are defined as the time dependent coupling coefficients between regions m and n for prompt and i-th group delayed neutrons, and $\Delta k^F_{mn}(t)$ and $\Delta k^A_{mn}(t)$ are the direct changes of the coupling coefficients due to the perturbation of δA and δF, respectively. The $l_m(t)$ and $\beta_{mn}(t)$ are the neutron generation time and delayed neutron fraction for i-th group, respectively. These parameters are defined by weighting with the inhomogenious importance function $G_{mg}(r)$ and neutron flux $\phi_g(r,t)$. The coupling coefficients, $k^p_{mn}(t)$ and $k^d_{imn}(t)$ mean the rate that a neutron born in region n produces fission neutrons in region m in the next generation, and their change is caused by the indirect change of flux distributions.

Integrating the precursor equation (2) over the volume of a region m, V_m, we obtain the equations for the precursor densities $C_{im}(t)$ of the delayed neutrons;

$$\frac{dC_{im}(t)}{dt} = \frac{1}{k_0}\beta_{im}(t)\ S_m(t) - \lambda_i C_{im}(t). \tag{7}$$

The $S_m(t)$ and $C_{im}(t)$ in Eqs. (6) and (7) are defined as

$$S_m(t) = \int_{V_m} dr F \phi_g(r,t) \tag{8}$$

and

$$C_{im}(t) = \int_{V_m} dr C_i(r,t). \tag{9}$$

When we choose the number of regions or reactors equal to one, i.e., N=1, the system becomes the conventional kinetics equations for one-point reactor. These multipoint kinetics equations can be regarded as the generalization of the conventional point kinetics equations. Now, Eqs. (6) and (7) can be solved by finite difference implicit scheme, Runge-Kutta methods, etc.

3. MODEL IMPLEMENTATION

A computer code has been developed using the model equations described in Section 2. The computational procedure is outlined.

(1) Time-independent steady state multigroup diffusion equation is solved for neutron flux distribution ϕ_g in the fine mesh domain of reactor core by the finite difference technique with the successive over-relaxation and cyclic Chebyshev acceleration. Correspondingly, the steady state precursor concentrations, C_i are evaluated in the same fine mesh domain.

(2) The steady state fission and precursor densities for each of N regions, $S_m(0)$ and $C_{im}(0)$ are evaluated using the fine mesh flux and precursor concentrations.

(3) The inhomogeneous importance functions, $G_{mg}(r)$ for each of N regions are calculated by solving Eq. (5) in the fine mesh domain. Note the equation system used is a fixed source inhomogeneous problem that can be easily solved.

(4) The coupling coefficients and kinetic parameters, k^f_{mn}, k^d_{imn}, Δk^f_{mn}, Δk^d_{imn}, l_{mn} and β_{mn} are calculated by integrating over regions, V_m and V_n using ϕ_g and G_{mg} evaluated in the fine mesh.

(5) Eqs. (6) and (7) are solved for time dependent fission density, $S_m(t)$ with a time step selected, Δt.

(6) The procedures (4) and (5) are repeated with time increment until the end of cauculation required. In this step, for incorporating the effect of parameters by the indirect change of flux distributions, the neutron flux $\phi_g(r,t)$ can be recalculated with the perturbed cross sections.

4. TEST RESULTS AND DISCUSSION

4.1 LMW LWR Test Problem

The first test has been carried out with the well known LMW LWR Benchmark problem[2]. The horizontal configuration of the one quadrant core of the Langenbuch-Maurer-Werner (LMW) 3-D test problem for LWR is given in Fig. 1. The core contains two groups of control rods initiating perturbations. Fig. 2 shows a side view of the core at the start and the end of the movement of the rod groups. Zero current boundary condition for inner surfaces representing symmetircally identical with other three quadrants and zero flux boundary condition on external surfaces are used. The transient consists in the withdrawal of the first group of control rods at a rate of 3 cm per second followed by insertion of the second control rods of diagonally opposite sites at the same speed. The resultant transient is followed for 60 seconds.

A uniform mesh consisting of 22, 22 and 10 mesh spacings in the x, y and z directions, respectively was used in solving the flux distribution for evaluating the coupling coefficients and kinetics parameters. Initial steady state is achieved by dividing the fission cross sections by the fundamental eigenvalues, and initial precursor concentrations are in equilibrium with the initial flux. The time step size of 0.5 seconds has been used. The uniform 24 coupled regions in the core were used for multi-point kinetics equations. The control rods are modeled by decreasing for the rod group 1 and then increasing for the rod group 2 the thermal removal cross section at a rate consistent with their velocity insertion. At any solution time step that the tip of the absorber being inserted does not coincide with a mesh line, the effect of the portion of the absorber existing past the given mesh line is smeared over the cell into which it projects by volume averaging absorber and cell properties.

Fig. 3 shows the behaviors of total mean power versus time. The result is in good agreement with the reference Benchmark solution. The result is also compared with the result from the CAE Electronics Ltd[4] by the coarse mesh method developed for CANDU simulator. It is observed that the present model yields very good result in this kind of slow transient problem carrying out asymmetric reactivity perturbation. The results compared with those by one point kinetics method also indicate that the present model yields superior result to the conventional point kinetics method.

4.2 CANDU Reactor Kinetic Benchmark Problem

The solution problem is a 3-D benchmark based on realistic three-region (reflector, inner and outer fuels) CANDU reactor model with zero flux boundary condition for external surfaces and on reactivity transients that represent the effects of loss of coolant followed by subsequent insertion of shutdown reactivity devices[3]. The problem is a typical CANDU fast transient problem with large asymmetric reactivity insertion requiring detailed 3-D analysis. The 3-D configuration of the reactor core with dimensions and material assignments is detailed in Figs. 4 and 5. The effect of loss of coolant is represented by linear decrease in the left-half core (5, 6, 10, 11, 17, 18, 22 and 23 in Fig. 4) thermal removal cross sections from 0. to 0.4 seconds, followed by a decrease in slower rate during the following 2.5 seconds. After a delay of 0.6 seconds, an incremental thermal removal cross section in upper right-half and down full core (shaded areas in Fig. 4 & 5) is added to simulate asymmetric insertion of shutdown devices at constant velocity in y-direction.

Non-uniform meshs consisting of 18, 18 and 10 mesh spacings in the x, y and z directions, respectively were used in solving the flux distributions for evaluating the coupling coefficients and kinetic parameters. The 72 coupled regions in the core were used for multi-point equations. For setting up coupled regions the core was divided by each one segment in the left and right outer core and two segments in inner core on x-direction, as much as the number of fine meshes for y-direction, and two segments on z-direction. Initial steady state is achieved by dividing the fission cross sections by the fundamental eigenvalues. The shutdown devices are modeled by increasing the thermal removal cross sections at a rate consistent with their top-to-bottom constant velocity insertion.

Fig. 6 shows the behaviors of total power versus time. The power behaviors after shutdown devices insertion differ significantly from the reference solution by the 3-D CANDU detailed kinetics code, CERBERUS. It is assumed that the observed difference is associated with the use of different absorber insertion models or with the inadequacy of the present approach using the reference core steady state flux distribution for evaluating the time-dependent coupling coefficients in this kind of fast large asymmetric reactivity insertion transient. However, the latter might be more contributive to this. Further study is required. Having resolved these differences, it is concluded that the solution are reliable for increasing power behaviors. Also, the results compared with those by one point kinetics method indicate that the present model yields superior results to the conventional point kinetics method. The CPU time required is about multi-points times as much as that of one point kinetics codes.

5. CONCLUSION

A computer code based on the coupled reactor kinetics theory has been developed. For the evaluation of its applicability, it was applied to typical 3-D reactor neutronic transient problems.

The test results showed that the present model yields reliable solutions for 3-D transient power behaviors with small computing time and produces superior results to the conventional point kinetics method.

It is generally observed that the present model produces accurate results for the slow or small reactivity transient behaviors even with asymmetric perturbations and there is accuracy sacrifice for the fast asymmetric large reactivity transient behavior. However, it is observed in some other tests that the accuracy can be improved by increasing the number of multi-point regions and using the optimised multi-region map. Another fundamental approach for improving accuracy is to update frequently the neutron flux distribution used in the evaluation of the coupling coefficients and kinetic parameters according to the severeness of the degree of local perturbation for incorporating the effects caused by the indirect change of flux distributions. But for doing this we have to pay for much computing time.

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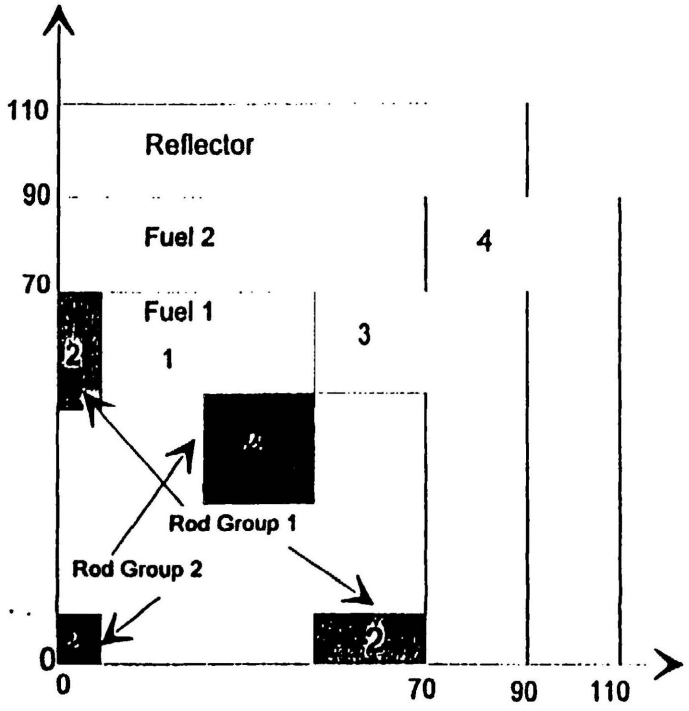


FIG. 1 LMW BENCHMARK : HORIZONTAL X-SECTION

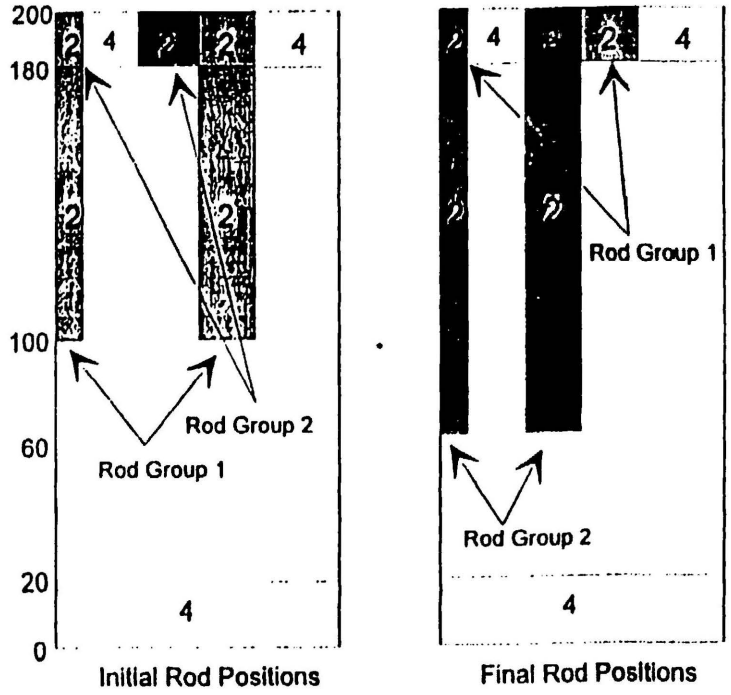


FIG. 2 LMW BENCHMARK : VERTICAL X-SECTION

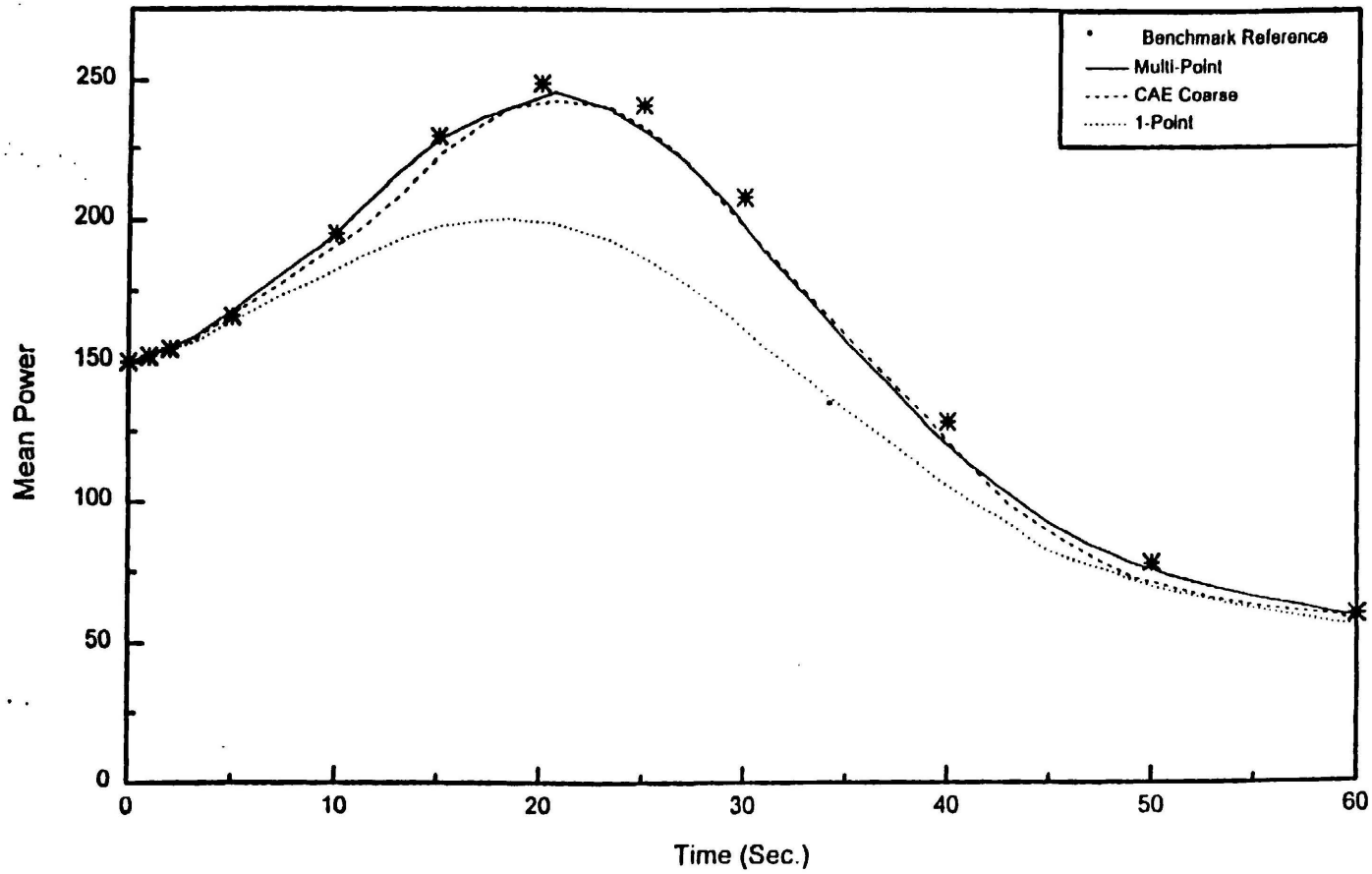
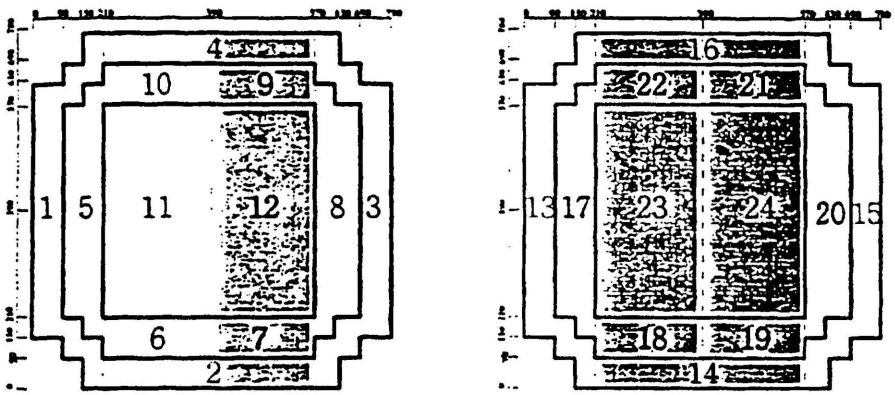


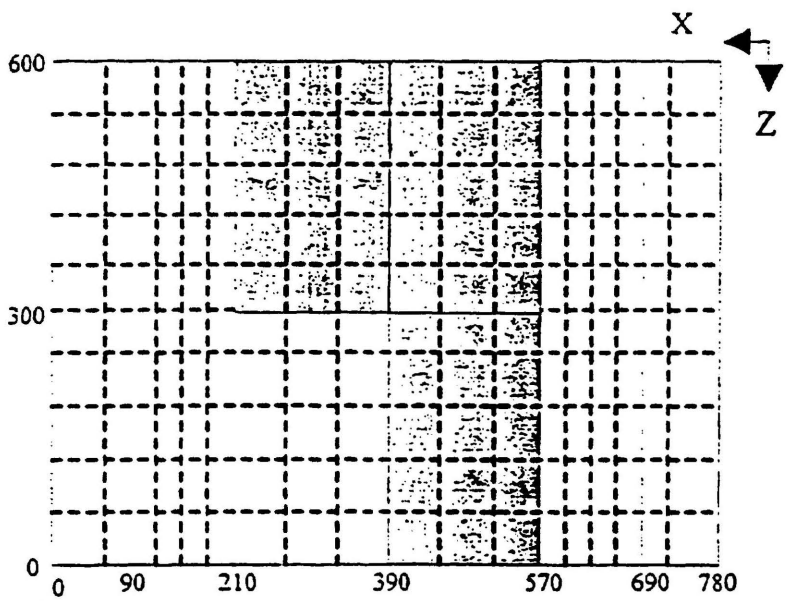
FIG. 6 MEAN POWER BEHAVIOR VERSUS TIME (LMW BENCHMARK PROBLEM)



Vertical Cross Section at Z=0 cm
Showing Region Assignment

Vertical Cross Section at Z=300cm
Showing Region Assignment

FIG. 4 CANDU BENCHMARK PROBLEM (X-Y DIRECTION)



Horizontal Cross Section at Y=390cm, Top View

FIG. 5 CANDU BENCHMARK PROBLEM (X-Z DIRECTION)

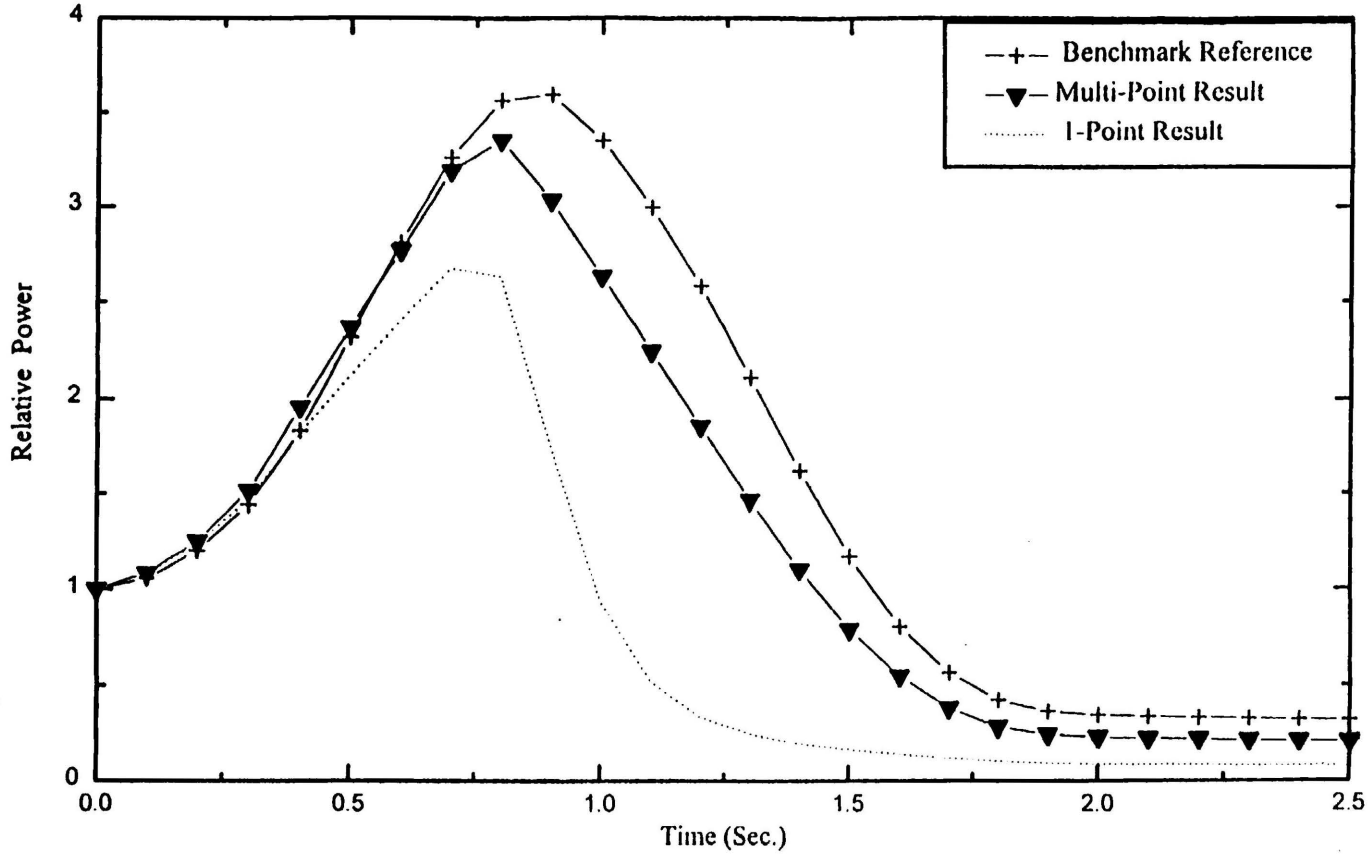


FIG. 6 RELATIVE POWER VERSUS TIME (CANDU BENCHMARK PROBLEM)