PREDICTION OF FAN ASSISTED FLOW IN A DUCT/PIPE NETWORK

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ABSTRACT

The commonly used fan+duct model is usually based on a table generated by matching the fan and system characteristic curves with the applied pressure drop across the fan+duct/pipe network and using linear or polynomial interpolation for intermediate values. However, this empirical approach can only handle a single system configuration for each table. If this approach is replaced by an algebraic formulation a general and flexible model can be developed. The algebraic model will be able to account for failure of resistances in the duct/pipe system as well as the failure of duct/pipe at an intermediate location. This paper presents the development of an algebraic model for fan+duct/pipe systems.

1 INTRODUCTION

A fan system may consist of any combination of duct/pipe elements, dampers, air cleaning elements, air heating elements, air conditioning elements, other equipments, and fans through which all or part of the air flow must pass. It can be an open system or a closed system. Closed systems form a loop and have no openings. Open systems have one or more air intake and one or more air discharge openings. Fan performance must match system requirements. The only possible operating point is at the location where the system characteristic intersect the fan characteristic. At such a point, the pressure developed by the fan exactly matches the system resistance and the flow through the system equals the fan flow rate. For an open system, the fan output must be enough to overcome the losses caused by the flow through the various system elements and to overcome any difference in kinetic energy from the system entrance to the system exit. A general equation for a fan with both open inlet and open outlet duct/pipe network, the fan total pressure is (Equation 19.8 of Reference 1):

Fan total pressure = entrance loss + inlet duct/pie loss + exit duct/pipe loss + exit loss

Fan performance in the first quadrant is commonly plotted with flow rate as abscissa and fan parameters as the ordinate. This is usually volume flow rate and pressure curve. The overall resistance to flow through a system will change with the flow rate. This variation can be shown graphically by plotting the pressure versus flow rate characteristics of the system in the same way the fan characteristics are presented. By matching these two, fan and system, characteristic with the applied pressure drop across the fan+duct/pipe network, a table of flow as a function of pressure drop can be developed. This is the commonly used fan+duct/pipe model in most computer programs. To achieve more accuracy, in many instances polynomial curves are fitted to represent flow as a function of pressure drop. In many situations, the fan curve in the first quadrant is not enough and is graphically extended to the second and fourth quadrant (Page 14-22 of Reference 1).

The above empirical approach relies on experiment and experience and provides very accurate model of fan+duct/pipe flow for a given configuration. However, this approach for fan+duct/pipe system modelling has one serious drawback. If there is any change in system configuration, for example a damper setting is changed, the previous flow versus pressure drop table is no longer valid and a new table must be generated. This can be avoided if the above tabular or polynomial model is replaced by an algebraic model based on a combination of theoretical

and empirical relationships. The algebric model also provides a more accurate or physical method for computing forced flow through a fan with duct/pipe system for flow forced with or against the fan. The algebric model is also able to account for failure of resistances in the duct/pipe system as well as the failure of duct/pipe at an intermediate location.

This paper presents the development of an algebraic model for fan+duct/pipe systems based on the fan design parameters and loss coefficients of various components of the duct network. In this model fan also act as a resistance to flow. In reality there are two components of fan resistance, constant resistance and variable resistance (Reference 1). The constant resistance is due to friction losses and variable resistance is due to the shock losses. In this development, the model is simplified and part of shock losses are ignored.

2 FAN MODEL

The fan laws are a particular version of the more general similarity laws that apply to all classes of turbo-machinery. They express the relationships among the performance variables for any two fans that have similar flow conditions. The variables include:

D	Fan size, m
Ν	Fan speed, rps
ρ	Fan fluid density, kg/m ³
Q	Fan flow rate, m ³ /s
Pft	Fan total pressure, pa
Pfv	Fan velocity pressure, pa
Pfs	Fan static pressure, pa
Pi	Fan input power, W
η	Fan total efficiency
η	Fan static efficiency
K _p	Compressibility coefficient
L,	Sound power level, dB

These parameters can be combined to form a number of dimensionless coefficients that are useful in fan engineering. For our purpose, the key dimensionless coefficients are, the flow coefficient, ϕ , and the pressure coefficient (total), Ψ_{T} .

flow coefficient, $\phi = Q K_p N^{-1} D^{-3}$	Equation-1
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pressure coefficient (total), $\psi_T = p_{ft} K_p \rho^{-1} N^{-2} D^{-2}$

For fans, dimensionless performance curves can be drawn using various coordinates. One combination is shown in Figure-1 (reproduced from page 12-29 of Reference 1). Flow coefficient is used as abscissa and pressure, power, efficiency, speed, diameter, and throttling coefficient are plotted as ordinate. The curve of our interest is the variation of ψ_{T} as a function of ϕ . In the range of ϕ from 0.35 to 0.8 the approximate relationship is:

$$\phi^2 = C_{f1} \left(C_{f2} - \psi_T \right)$$
 Equation-3

where C_{f1} and C_{f2} are two constant numbers.

A comparision of the relationship given by Equation-3 with the values given in Figure-1 is shown in Figure-2. The relationship shows significant deviation for the range of 0 from 0.0 to 0.35. This deviation is due to the assumption related to shock losses. The head developed by the fan is more or less constant but due to friction

losses and shock loses the final head is much lower. The decrease in the final head due to friction is proportional to the square of flow. The decrease in final head due to shock increases as we move away from the design point. We assumed that the shock losses behave similar to the friction losses, which is not true for flows less than design flows.

Let us define

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$$C_{\rm E} = C_{\rm f3} K_{\rm p} \rho^{-1} N^{-2} D^{-2}$$
Equation-4

Substituting the definitions of coefficients from Equations-1 -2 and -4 into Equation-3:

$$Q^{2} K_{p}^{2} N^{2} D^{6} = C_{f1} (C_{f3} - p_{f1}) K_{p} \rho^{-1} N^{-2} D^{-2}$$

$$Q^{2} = C_{f1} D^{4} K_{p}^{-1} \rho^{-1} (C_{f3} - p_{f1})$$
Equation-5

at maximum flow, Equation-5 is:

$$Q_{max}^{2} = C_{f1} D^{4} K_{p}^{-1} \rho^{-1} C_{f3}$$
 Equation-6

There is a relation ship defined for a given fan design between the design flow and maximum flow:

$$Q_{max} = C_{f4} Q_{design}$$
Equation-7

Substituting Equation-7 into Equation-6:

$$C_{f4}^{2} Q_{design}^{2} = C_{f1} D^{4} K_{p}^{-1} \rho^{-1} C_{f3}$$

$$D^{4} K_{p}^{-1} \rho^{-1} = C_{f4}^{2} C_{f1}^{-1} C_{f3}^{-1} Q_{design}^{2}$$
Equation-8

A relationship can also be derived between design pressure and other constants from Equation-5:

$$Q^{2}_{design} = C_{f1} D^{4} K_{p}^{-1} \rho^{-1} (C_{f3} - p_{f1,design})$$
Equation-9

Substituting Equation-6 into Equation-9

$$Q^{2}_{dessgn} / Q^{2}_{max} = (C_{f3} - p_{fLdesign}) / C_{f3}$$

$$C^{2}_{f4} = 1 - p_{fLdesign} / C_{f3}$$

$$C_{f3} = (1 - C^{2}_{f4})^{-1} p_{fLdesign} = C_{f5} p_{fLdesign}$$
Equation-10

Substituting Equations 8 and 10 in Equation 5:

$$Q^{2} = (C_{f4}^{2} / C_{f5}) (Q_{design}^{2} / p_{fLdesign}) (C_{f5} p_{fLdesign} - p_{ft})$$
Equation-11

The Equation-11 is validated against the ventilation outlet curve supplied by Sheldon Engineering Ltd. and reproduced in Figure-3. From Figure-3 the maximum flow is 18,000 CFM. Substituting the values of various constants in Equation-11 (where Q is in CFM and p_{fi} in wg):

$$Q^2 = 26193182 (12.375 - p_{f})$$
 Equation-12

The following table compares the calculated flow and flow given in Figure-3 for various fan total head (pressure).

P _{ft} in wg	Q _{manufacturer} CFM	Q _{calculated} CFM
0	18000	18004
2	16500	16485
4	15200	14811
6	13600	12922
8	11600	10705

3 CALCULATION OF PARAMETERS FOR FAN MODEL

The Equation-11 can be used to define the fan curve if the fan design pressure and fan design flow is known. However in many situations, the design flow and fan size is the only known quantity. Under this situation, a value of design pressure can be calculated from a modified form of Equation-6:

$$Q_{max}^2 = C_{f1} D^4 K_p^{-1} \rho^{-1} C_{f3}$$
 Equation-6

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Substituting Equations 7 and 10 into Equation 6:

$$C_{f4}^{2} Q_{design}^{2} = C_{f1} D^{4} K_{p}^{-1} \rho^{-1} C_{f5} p_{ft.design}$$

$$p_{ft.design} = (C_{f4}^{2} C_{f1}^{-1} C_{f5}^{-1}) Q_{design}^{2} \rho^{2} D^{-4} K_{p}^{-1}$$
Equation-13

Under the worst situation, the only information available is the design flow rate. To define the design pressure we have to use similarity laws of fan design, also known as the fan laws. The "Fan Law #3b", given on page 12-3 of Reference 1, is most useful for this situation and can be derived from the Equation-6:

$$Q_{max, fanl}^{2} = C_{fl} D_{fanl}^{4} K_{p, fanl}^{-1} \rho_{fanl}^{-1} C_{f3}$$

$$C_{f4}^{2} Q_{design, fanl}^{2} = C_{fl} D_{fanl}^{4} K_{p, fanl}^{-1} \rho_{fanl}^{-1} C_{f5} p_{fl,design, fanl}$$
Equation-14
$$C_{f4}^{2} Q_{design, fan2}^{2} = C_{f1} D_{fan2}^{4} K_{p, fan2}^{-1} \rho_{fan2}^{-1} C_{f5} p_{fl,design, fan2}$$
Equation-15

Therefore

$$(p_{fLdesign, fanl}/p_{fLdesign, fan2}) = (Q_{design, fanl}/Q_{design, fan2})^2 (D_{fanl}/D_{fan2})^4 (K_{p, fan1}/K_{p, fan2})^1 (\rho_{fan1}/\rho_{fan2})^i$$
Equatin-16

assuming the values of K_p and ρ (for same gas) are identical or similar for both fans

$$(p_{fLdesign, fan1}/p_{fLdesign, fan2}) = (Q_{design, fan1}/Q_{design, fan2})^2 (D_{fan1}/D_{fan2})^4$$
Equation-17

If the values of $p_{fi,design, fan2}$, $Q_{design, fan2}$ and D_{fan2} for a standard fan are known and $Q_{design, fan1}$ and D_{fan1} is known then pfildesign, fanl can be calculated. The Dfan is the impeller diameter. Based on geometric similarity laws:

$$(D_{fan1}/D_{fan2})_{impeller} = (D_{fan1}/D_{fan2})_{ivpical} = (D_{fan1}/D_{fan2})_{inlet}$$
Equation-18

Once the values of pfildesign and Qdesign are known, and assuming that the shock losses are less than the friction losses for the flow regime of our interest, Equation-11 can be used to calculate the flow. It should be noted that Equation-11 calculates the square of volumetric flow and the flow is square root of the absolute value of the calculated value with the flow direction determined by the sign of $(C_{15} p_{fl,design} - p_{fl})$.

CONVERTING FAN MODEL TO STANDARD FLOW THROUGH A DUCT/PIPE NETWORK MODEL 4

The fans which needs to be modelled are used with a network of duct or pipes for air inlet and outlet. To use the above fan model with the duct/pipe flow model, Equation-11 should be modified to the duct flow model given by:

$$Q^2 = (2 A^2 \Delta p) / (p K)$$

where

Α	Duct/pipe area
ρ	fluid density
Δp	pressure difference
K	Loss coefficient

If we write Equation-11 as:

$$Q^{2} = [2 A^{2} (C_{f5} p_{ft,design} - p_{ft}) / \rho] [(C_{f4}^{2}/C_{f5}) (Q_{design}^{2} / p_{ft,design}) \rho / (2 A^{2})]$$

and define

$$\Delta p_{far.} = (C_{f5} p_{f.design} - p_{ft})$$
Equation-20
$$K_{fan} = [(2 A^2 p_{f.design}) / ((C_{f4}^2/C_{f5}) Q_{design}^2 \rho)]$$
Equation-21

then the fan equation becomes

$$Q^{2} = (2 A^{2} \Delta p_{fan}) / (\rho K_{fan})$$
Equation-22

For ventilation outlet fan, 0.762 m inlet diameter (0.456 m² area), 3.54 m³/s design flow and 2.465 kPa design head and air density 1.29 kg/m³, the value of K_{fan} is 13.7556 and C_{f5} is 1.25.

The advantage of defining the fan model in terms of fan loss coefficient is that for similar designs the value of fan loss coefficient is constant.

$$K_{fan,1} = \left[2 A_1^2 p_{ft,design,1} Q_{design,1}^2 \rho_1^{-1} (C_{f4}^2/C_{f5})^{-1}\right]$$
Equation-21

Equation-19

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replacing fan design pressure using Equation-16 and assuming that fluid density is same:

$$K_{fan.1} = \left[2 A_{1}^{2} (D_{fan1}/D_{fan2})^{-4} p_{ft.design. fan2} (Q_{design. fan1}/Q_{design. fan2})^{2} Q_{design.1}^{-2} \rho_{1}^{-1} (C_{f4}^{2}/C_{f5})^{-1}\right]$$

$$K_{fan.1} = \left[2 A_{2}^{2} p_{ft.design.2} Q_{design.2}^{-2} \rho_{2}^{-1} (C_{f4}^{2}/C_{f5})^{-1}\right]$$

$$K_{fan.1} = K_{fan.2}$$

5 MODEL FOR FAN IN A DUCT/PIPE NETWORK

The fan is not the only component in a flow path. There are pipes/ducts before and after a fan. The pressure difference known or to be calculated is across the total duct/pipe network. If we define:

Pressure at the pipe inlet Pi pressure at the pipe outlet p_o Pressure at the fan inlet Pfi pressure at the fan outlet Pf.o Fan total pressure = $(p_{f,o} - p_{f,i})$ Pft Total Loss coefficient in the inlet pipe K, K, Total Loss coefficient in the outlet pipe Total Loss coefficient in the inlet and outlet pipe K_{pipe} Pressure drop in the inlet pipe = $(p_i - p_{f,i})$ Δp_i Pressure drop in the outlet pipe = $(p_{f,o} - p_o)$ Δp_o Overall Pressure drop = External pressre across duct/pipe network = $(p_i - p_o)$ Δp

Then

$$\Delta p = (p_i - p_o) = (p_i - p_{f,i}) + (p_{f,i} - p_{f,o}) + (p_{f,o} - p_o) = +\Delta p_i - p_{f,i} + \Delta p_o$$
Equation-24
where
$$\Delta p_i = Q^2 \rho K_i / (2 A^2)$$
Equation-25
Equation-26

because (see Equation-20)

$$\Delta p_{fan} = C_{f5} p_{fldesign} - p_{fl} = Q^2 \rho K_{fan} / (2 A^2)$$

therefore,

$$\mathbf{p}_{ft} = \mathbf{C}_{f5} \mathbf{p}_{ft,design} - \mathbf{Q}^2 \mathbf{\rho} \mathbf{K}_{fan} / (2 \mathbf{A}^2)$$
Equation

Substituing the Equations 25, 26 and 27 for Δp_i , p_{fi} , Δp_o into Equation-24:

$$\Delta p = Q^{2} \rho K_{i} / (2 A^{2}) - C_{fS} p_{ft.design} + Q^{2} \rho K_{faa} / (2 A^{2}) + Q^{2} \rho K_{o} / (2 A^{2})$$

$$C_{fS} p_{ft.design} + \Delta p = Q^{2} \rho (K_{i} + K_{fan} + K_{o}) / (2 A^{2})$$

$$Q^{2} = [2 A^{2} (C_{fS} p_{ft.design} + \Delta p)] / [\rho (K_{pipe} + K_{fan})]$$

n-27

Equaqtion-23

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Equation-26

where:

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Flow in a pipe/duct network
Duct/Pipe area
Fan design pressure head
external pressure drop across duct/pipe network =
(inlet pressure - outlet pressure)
Gas density
Total loss coefficient in the inlet and outlet pipe/ductwork
Fan loss coefficient

Equation-28 defines the fan assisted flow in a pipe/duct network and is suitable for defining fan model in a numerical solution scheme.

To test the validity of this model for Point Lepreau normal operation we need the values of various components which are part of the pipe loss coefficient. Starting at the inlet of ventilation inlet system to the outlet of ventilation outlet. The pressure drops are assumed to be:

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Inlet Roll type pre-filter = 0.4 in wg = 99.6 Pa
Inlet fine filter = 0.4 in wg = 99.6 Pa
Inlet ducting losses = 1.8 in wg = 448.2 Pa
Outlet ducting losses = 1.0 in wg = 249 Pa
pre filter = 0.4 in wg = 99.6 Pa
First HEPA = 1.0 in wg = 249 Pa
Charcoal filter = 2.3 in wg = 572.7 Pa
Second HEPA = 1.0 in wg = 249 Pa
Ducting after filter = 0.3 in wg = 74.7 Pa
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using the duct flow equation and assuming normal operation flow of 4.72 m³/s (10000 scfm), 0.762 m base pipe diameter (0.456 m² area) and air density 1.29 kg/m³ the pressure drop across various components can be changed to equivalent loss coefficient, K, for that component.:

 $K = 2 A^2 \Delta p / \rho Q^2 = [2 (0.456 0.456)/(1.29 4.72 4.72)] \Delta p = 0.01447 \Delta p$

Inlet Roll type pre-filter = 1.441 Inlet fine filter = 1.441 Inlet ducting losses = 6.485 Outle: ducting losses= 3.606 pre filter = 1.441 First HEPA = 3.606 Charcoal filter = 8.287 Second HEPA = 3.606 Ducting after filter = 1.081

 $\mathbf{K}_{\mathsf{pipe}} = (1.441 + 1.441 + 6.485) + (3.606 + 1.441 + 3.606 + 8.287 + 3.606 + 1.081) = 30.994$

$$K_{fan} = 13.756, C_{f5} = 1.25, p_{iLdesign} = 2.465$$
 kPa for ventilation outlet fan.

$$Q^{2} = \left[2 A^{2} \left(C_{f5} p_{fi,design} + \Delta p\right)\right] / \left[\rho \left(K_{pipe} + K_{fan}\right)\right]$$

 $Q^2 = [2 (0.456)^2 (1.25 2465 + 0)] / [1.29 (30.994 + 13.756)] = 22.197$

 $Q = 4.711 \text{ m}^3/\text{s}$

The predicted flow is very close to design specified flow of 4.72 m³/s.

6 MODEL FOR PARALLEL FANS IN A DUCT/PIPE NETWORK

 $\Delta p = (p_i - p_o) = (p_i - p_{f_i}) + (p_{f_i} - p_{f_o}) + (p_{f_o} - p_o) = +\Delta p_i - p_{f_i} + \Delta p_o$

In many situation there are two pumps in parallel to ensure that there is a backup fan available. If both of these fans are operating then each fan is handling half the duct/pipe network flow.

where

$$\begin{aligned} \Delta p_{i} &= Q^{2} \rho K_{i} / (2 A^{2}) \\ \Delta p_{o} &= Q^{2} \rho K_{o} / (2 A^{2}) \\ \Delta p_{fan} &= C_{f5} p_{fLdesign} - p_{ft} = (Q/2)^{2} \rho K_{fan} / (2 A^{2}) \\ \Delta p &= Q^{2} \rho K_{i} / (2 A^{2}) - C_{f5} p_{fLdesign} + Q^{2} \rho \{K_{fan}/4\} / (2 A^{2}) + Q^{2} \rho K_{o} / (2 A^{2}) \end{aligned}$$

thus

$$Q^{2} = \left[2 A^{2} \left(C_{fs} p_{ftdesign} + \Delta p\right)\right] / \left[\rho \left(K_{pipe} + K_{far}/4\right)\right]$$
Equation-29

To test the validity of this model, maximum ventilation flow for the Point Lepreau station is calculated. The maximum flow will occur when both fans are operating and the ventilation outlet filter train is by-passed.

$$K_{pipe} = (1.441+1.441+6.485)+(3.606+1.081) = 14.054$$

$$K_{far}/4 = 13.756/4 = 3.439$$

$$Q^{2} = [2 (0.456)^{2} (1.25 2.465 1000 + 0)] / [1.29 (14.054 + 3.439)] = 56.785$$

$$Q = 7.536 \text{ m}^{3}/\text{s} = 27,128 \text{ m}^{3}/\text{hr}$$

The predicted flow is very close to the approximate flows given as 25,000 m³/hr in the design description of this system.

7 PREDICTION OF PRESSURES DIFFERENTIALS ACROSS VARIOUS COMPONENTS

The pressure differentials across various duct/pipe network components except fan can be calculated from the equation:

$$\Delta p_{\text{components}} = Q^2 \rho K_{\text{components}} / (2 \text{ A}^2)$$
Equation-30

and for fan:

$$p_{ft} = C_{fS} p_{tt,design} - Q^2 \rho K_{fan} / (2 A^2)$$

Equation-31

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Substituting the value of $Q^2 \rho / (2 A^2)$ form Equation-28:

$$\Delta p_{\text{components}} = K_{\text{components}} \left[(C_{f5} p_{\text{ft,design}} + \Delta p) / (K_{\text{pipe}} + K_{\text{fan}}) \right]$$
Equation-32

and for fan:

$$p_{ft} = C_{f5} p_{ft,design} - K_{fan} \left[(C_{f5} p_{ft,design} + \Delta p) / (K_{pipe} + K_{fan}) \right]$$
Equation-33

Either the Equations 30 and 31 or Equations 32 and 33 can be used to predict the pressure differentials across various components.

8 PREDICTION OF LOSS COEFFICIENT OF DUCT/PIPE NETWORK

In the example given in Section 5, the loss coefficient of almost all components in the flow loop was known. Under most situations, these values will not be known. During normal operation, operator will change the damper settings to achieve the desired flow and pressure drops. The loss coefficient varies as the setting of dampers are changed. Equations need to be developed to calculate the loss coefficient from the given normal operation flow and pressure differences. The basic equation for pipe/duct network without fan and with fans are:

for basic pipe network

$$K_{\text{pipe network}} = (2 \text{ A}^2 \Delta p_{\text{pipe network}}) / (\rho \text{ Q}^2)$$

for pipe network with fan

$$K_{\text{nume network}} = [2 A^2 (C_{15} p_{\text{it design}} + \Delta p)] / [\rho Q^2] - K_{\text{fan}}$$

9 PROGRAM FANCONST

This report provides a large number of equations which can be used to calculate the value of fan design pressure and total duct loss coefficient with loss coefficient of major components from the given design flow and pressure drop values. The program FANCONST consolidates these equations in a FORTRAN program. Three situation of fan information availability are considered: 1) Fan design pressure and inlet diameter is known. 2) Fan design flow and inlet diameter is known and the fan type is similar to the Point Lepreau ventilation inlet fan (0.762 m inlet diameter, 3.54 m^3 /s design flow and 2.465 kPa design pressure). 3) Fan design flow and inlet diameter is known and the fan type is similar to a fan whose inlet diameter, design pressure and design flow is supplied. Based on supplied pressures at the duct inlet and outlet and steady state flow at that pressure difference, the total loss coefficient for duct/pipe network are calculated. Based on pressure drop across major component at that steady state flow, the loss coefficient of those major components are calculated.

Equation-35

10 PROGRAM FANFLOW

The fan in a duct+pipe network model given in this report is converted to a stand alone FORTRAN program, FANFLOW, to generate flow versus pressure drop tables for various system configuration. We used the FANFLOW programm to generate the ventilation outlet fan curve by assuming no ducting before are after the filter housing. The predicted curve is compared with the fan curve supplied by the manufacturer in Figure-4. The agreement for flow greater than the design flow $(3.54 \text{ m}^3/\text{s})$ is reasonable.

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REFERENCES

1 R. Jorgensen, "Fan Engineering: An Engineer's Handbook of Fans and Their Applications"', Buffalo Forge Company, Buffalo, New York.



Figure-1 Dimensionless Fan Performance Curve

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Dimentionless Fan Performance Curve

Figure-2 Predicted Dimensionless Fan Performance Curve



Figure-3 Fan Performance Curve for Ventilation Outlet Fan



Ventilation Outlet Fan Curve

Figure-4 Predicted Fan Curve for Ventilation Outlet Fan

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