AN AUTOMATED PROCEDURE FOR EXCLUDING READINGS OF FAILED DETECTORS IN FLUX MAPPING
by

B. Rouben, H.W. Dassen and K. Ossia
AECL CANDU
Sheridan Park Research Community
Mississauga, Ontario
L5K 1B2

ABSTRACT

The conceptually correct way of handling the readings of failed detectors in flux mapping is to exclude them from the numerical procedure.

An automated way to exclude such irrational readings is implemented in the off-line version of the flux-mapping program. Its performance is illustrated by means of tests with various numbers of irrational readings.

Results demonstrate that the proposed method can credibly replace the existing approximate procedure in the on-line program version as well, if a modern-day computer is used.

INTRODUCTION

Background

Flux mapping is a well-known procedure for synthesizing a three-dimensional flux distribution inside the reactor core by expansion in a set of predetermined flux shapes ("modes"), using the readings of in-core vanadium detectors as data. The amplitudes of the various modes used in the expansion are determined by a least-squares fit of the synthesized flux to the detector readings.

Present Work

The present work extends the capability of the off-line flux-mapping program to correctly exclude the readings of failed detectors from the numerical procedure, and compares results obtained with the exclusion method from those obtained with the existing approximate substitution method.

NUMERICAL BASIS OF FLUX MAPPING

Modal Expansion

Assume that the thermal flux at any point $\vec{r}$ in the core, $\phi(\vec{r})$, can be expressed as a linear combination of pre-calculated flux modes $\psi_n(\vec{r})$:

$$\phi(\vec{r}) = \sum_{n=1}^{N} A_n \psi_n(\vec{r}) \tag{1}$$

where N is the total number of modes used and A_n is the amplitude of the nth mode.

The flux-mapping procedure determines the mode amplitudes from the readings of in-core detectors, and then uses Equation (1) to calculate the three-dimensional flux distribution in the core. The algorithm is described below.

Suppose there are D in-core detectors at positions labelled $\vec{r}_d$, d=1, ..., D.

Writing Equation (1) for the special case of these detectors,

$$\phi(\vec{r}_d)=\sum_{n=1}^{N} A_n \psi_n(\vec{r}_d) \qquad d=1,\ \ldots,\ D \qquad (2)$$

The values $\phi(\vec{r}_d)$, calculated from Equation (2) once the mode amplitudes are known, are called the mapped detector fluxes.

Equation (2) can be rewritten in the form:

$$\phi_d=\sum_{n=1}^{N} A_n M_{dn} \qquad (3)$$

where $$\phi_d=\phi(\vec{r}_d) \qquad (4)$$

and $$M_{dn}=\psi_n(\vec{r}_d) \qquad (5)$$

In turn, Equation (3) can be written in matrix form:

$$\phi=M\cdot A \qquad (6)$$

where $$A=\begin{pmatrix} A_1 \\ : \\ A_N \end{pmatrix} \qquad (7)$$

is the Nx1 vector of amplitudes,

$$\phi=\begin{pmatrix} \phi_1 \\ : \\ \phi_D \end{pmatrix} \qquad (8)$$

is the Dx1 vector of mapped detector fluxes, and

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1N} \\ M_{21} & & & \\ \vdots & & & \\ M_{D1} & M_{D2} & \cdots & M_{DN} \end{pmatrix} \tag{9}$$

is the DxN matrix of mode values at the detector positions.

Solving for the Mode Amplitudes

The detector fluxes at some particular time are assumed known from site measurements. The measurements (readings) are essentially electric currents generated by the in-core detectors, and depend on the flux at the detectors and on the detector sensitivity.* The readings are converted to fluxes by dividing by the detector sensitivity factors:

$$F_d = \frac{E_d}{S_d} \equiv K_d E_d \qquad d=1,\ \ldots,\ D \tag{10}$$

where E_d is the reading for detector d

$K_d \equiv \dfrac{1}{S_d}$ is the inverse sensitivity of detector d

and F_d is the derived "measured flux" (also sometimes called the "calibrated flux") for detector d.

The measured fluxes are denoted collectively by the Dx1 vector:

$$F = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_D \end{pmatrix} \tag{11}$$

The aim of flux mapping is to determine the amplitudes A_n to obtain the best fit of the mapped fluxes ϕ_d to the measured fluxes F_d.

Generally, there are many more detectors than modes, i.e., D > N. For example, in the CANDU 6 there are 102 in-core detectors, i.e., D=102, and the

* In fact, the detector lead cable also contributes to the current, and must be taken into account. This does not change the procedure described here in a material way.

number of modes used in the flux-mapping expansion, N, ranges between 15 and 28.

Since it is impossible (in general) to obtain a perfect fit to D detector fluxes using a smaller number N of unknowns A_n, the flux-mapping method obtains a least-squares fit of the mapped fluxes ϕ_d to the measured fluxes F_d.

Define a weighted sum of squares of differences between the mapped and measured fluxes:

$$\epsilon=\sum_{d=1}^{D} w_d^2(\phi_d-F_d)^2 \qquad (12)$$

where the w_d are selected weight values.

Using Equation (3) for the mapped fluxes, the sum of squares is written as a function of the mode amplitudes:

$$\epsilon=\sum_{d=1}^{D} w_d^2\left\{\sum_{n=1}^{N} M_{dn}A_n-F_d\right\}^2 \qquad (13)$$

The sum of squares is minimized, i.e., a least-squares fit is achieved, by imposing the condition for an extremum, i.e.,

$$\frac{\partial\epsilon}{\partial A_n}=0 \qquad n=1,\ \ldots,\ N \qquad (14)$$

Using Equation (13), this condition can be written:

$$2\sum_{d=1}^{D} w_d^2\left\{\sum_{k=1}^{N} M_{dk}A_k-F_d\right\}M_{dn}=0 \qquad n=1,\ \ldots,\ N \qquad (15)$$

which can again be rewritten

$$\sum_{d=1}^{D} w_d^2\left\{\sum_{k=1}^{N} M_{dk}A_k\right\}M_{dn}=\sum_{d=1}^{D} w_d^2F_dM_{dn} \qquad n=1,\ \ldots,\ N \qquad (16)$$

Using the Dx1 vector of weights

$$W = \begin{pmatrix} w_1 \\ \vdots \\ w_D \end{pmatrix} \tag{17}$$

and its 1xD transpose

$$W^T = (W_1, \ \ldots \ W_D) \tag{18}$$

as well as the transpose M^T of M, Equation (16) can be cast in matrix form:

$$M^T \cdot (W \cdot W^T) \cdot M \cdot A = M^T \cdot (W \cdot W^T) \cdot F \tag{19}$$

Inverting this equation, the amplitude vector is obtained as

$$A = H \cdot F \tag{20}$$

where the NxD "pseudo-inverse" matrix H is given by:

$$H = \{M^T \cdot (W \cdot W^T) \cdot M\}^{-1} \cdot M^T \cdot (W \cdot W^T) \tag{21}$$

Once the modes-at-detectors matrix M has been computed and the weight vector W has been chosen, the matrix H can be calculated by inversion (i.e., Equation (21)), and the amplitudes A_n can be determined by a simple matrix multiplication, Equation (20).

Equation (1) can then be used to calculate the full three-dimensional flux distribution in the core, and Equation (2) or (3) provides the mapped detector fluxes.

An important by-product of the calculation is the zone-flux distribution. The reactor core is subdivided into a number of control zones (14 in the CANDU 6). The average values of mapped flux in the 14 zones are used by the regulating system to control the light-water fills in the zone-control compartments. It is therefore particularly important that the flux mapping reproduce zone fluxes accurately, i.e., to better than 0.5 to 1.0%, or about 1 MW (in round numbers) in equivalent power units.

FLUX MAPPING WITH FAILED DETECTORS

Occasionally, the in-core flux-mapping detectors fail or "go irrational". Detectors are deemed to have failed according to two distinct criteria:

- the measured readings are a priori identified as irrational - that is, they fall outside a reasonable range of values, or
- after the flux map has been calculated, the difference between the mapped and measured fluxes is found to be too large (typically, greater than 15% of the fundamental mode amplitude).

When irrational detectors have been identified, the conceptually correct way of treating them is to consider them as unreliable data, and exclude them from the numerical procedure to calculate the amplitudes.

Existing (Substitution) Procedure for Handling Failed Detectors

The present version of the flux-mapping program instead handles irrational detectors by substituting for the corresponding fluxes best-estimate values. This substitution method allows irrational detectors to be kept in the least-squares sum of Equation (15) without invalidating the procedure. The best-estimate detector fluxes can, for instance, be computed at first from the fundamental mode used in the mapping. Subsequently, once the first flux map has been obtained, the best-estimate fluxes can be taken as the mapped detector fluxes.

In the framework of this substitution method, the flux mapping is repeated iteratively. At each pass, irrational detector readings are replaced as explained above, until no new irrationalities are identified. A weakness that remains, however, is that there is no check on the convergence of the mapped fluxes before the iterations are stopped. Thus, the process can end with non-converged mapped values of flux.

New (Exclusion) Procedure for Handling Failed Detectors

While it is practical, the substitution method described above is clearly an approximation, substituting calculated values for measured values.

As stated earlier, once a detector has been established as irrational, it should, as a matter of principle, simply be dropped from the flux-mapping process. That is, this detector should be taken out of the least-squares sum. In this way, no "best-estimate" value can skew the results.

Excluding failed detectors from the least-square sum, however, has so far been impossible in the on-line version of the flux-mapping program. The reason for this is that the on-line program uses a precalculated "pseudo-inverse" matrix $\boldsymbol{H}$ to produce the least-squares fit. The dimensions of this matrix are preset and cannot be changed to accommodate varying numbers of detectors.

The restrictions indicated in the previous paragraph do not, however, apply to the off-line version of the flux-mapping program. In the present work, this has been used to advantage to re-program the process. In the version developed here, failed detectors are excluded from the mapping, and the matrices are reduced to a dimension equal to the number of "good" (non-irrational) detectors.

The special handling of failed detectors is completely automated so that the program user need not intervene by "manually" removing readings of the failed detectors from the data set. Detectors are re-ordered internally so that the valid detector readings form a contiguous set, permitting an efficient way of calculating the new "pseudo-inverse" matrix $\boldsymbol{H}$ for the reduced set. The detector re-ordering is then "unfolded". The mapped fluxes at the irrational detectors are still generated for information purposes, although they do not enter into the least-squares process.

With this exclusion method, iterative passes are again performed whenever the flux mapping uncovers new irrational detectors (through large differences between mapped and measured fluxes). However, when new irrationalities are no longer present, the mapped fluxes are automatically converged, in contrast to the substitution method.

RESULTS

A typical CANDU-6 flux-mapping calculation is investigated. As mentioned earlier, in the CANDU 6 reactor there are 102 flux-mapping detectors. In normal operation, with reactivity devices in their nominal position (i.e., adjusters in core, mechanical control absorbers out of core), a set of 15 flux-mapping modes, listed in Table 1, is used.

The study began with a typical set of detector readings, with no irrational detectors. The flux mapping in this situation provided a good fit to the detector fluxes (standard deviation of differences between mapped and measured fluxes = 2.81%).

To investigate the effect of failed detectors, a progressively greater number of detector readings were replaced at random by irrational (too high or too low) values.

Results obtained with the existing substitution method and the exclusion method are compared in Table 2. Results shown include the standard deviation of percent differences between mapped and measured (calibrated) fluxes, the largest value of these individual differences, the largest difference in the zone-averaged fluxes (in equivalent MW units) and the number of zones for which the difference is greater than 1 MW.

It is clear from the results in Table 2 that the existing substitution method is, in practice, adequate when only a few (<10) detectors are failed. As the number of irrational readings increases beyond 10, the quality of fit deteriorates significantly: the standard deviation of the differences between mapped and calibrated fluxes increases beyond 3%, and the largest individual difference can exceed 10%. Of even greater significance, the zone-flux error (as reckoned from the difference with the "correct" exclusion method zone flux) is greater than 1 MW for several (typically 5 to 10) zones.

Furthermore, when 10 or more irrational detectors are present, the existing substitution method often leaves a relatively large number of mapped detector fluxes (20 or more) with a convergence level poorer than 0.5%. This can detract significantly from the quality of the three-dimensional mapped flux distribution in the core.

From the perspective of both individual detector fluxes and the three-dimensional flux distribution (including the zone fluxes), the existing substitution method is thus seen to be potentially unreliable when more than a few detectors are irrational.

As is evident from Table 2, the quality of fit with the exclusion method remains excellent ($\sigma \approx 2.6$-2.9%), regardless of the number of failed detectors. The exclusion method, in addition, does not suffer from convergence difficulties, as earlier explained. The method is therefore generally more stable than the substitution method.

The CPU time per flux-map execution including recalculation of the pseudo-inverse matrix, is only about 0.4 s on an Apollo DN10000 computer, and is not very dependent on the number of failed detectors. The proposed exclusion method can therefore credibly be implemented in the on-line version of the flux-mapping code if a modern computer with sufficient central memory and execution speed is used.

SUMMARY

An automated method for excluding irrational detector readings from the flux-mapping procedure was successfully implemented in the off-line version of the flux-mapping program.

This method has the advantage that it is conceptually the correct one to use whenever a detector is failed: quite simply, failed detectors should not be retained within the flux-mapping scheme.

Numerical advantages of the new algorithm become more evident as the number of failed detectors increases, especially as the number of failures increases beyond 10.

Although the new procedure requires a recalculation of a new pseudo-inverse matrix whenever an additional detector failure is encountered, the extra computing is not significant for present-day computers, and is recommended for any off-line version of the flux-mapping code. The execution time on modern-day computers is short enough (<0.5 s) to allow implementation even on the on-line program version.

Table 1 15-Mode Set Used in Flux Mapping	
Mode Number	Description of Flux Shape
1	Time-average flux distribution
2	First azimuthal (A)
3	First azimuthal (B)
4	First axial
5	Second azimuthal (A)
6	Second azimuthal (B)
7	First azimuthal x first axial (A)
8	First azimuthal x first axial (B)
9	First radial x second axial (A)
10	First radial x second axial (B)
11	First azimuthal x second axial (A)
12	First azimuthal x second axial (B)
13	Third azimuthal (A)
14	Third azimuthal (B)
15	Second radial Bessel function

Table 2
Comparison of Substitution (S) and Exclusion (E) Methods

Number of Initial Irrational Detectors	Standard Deviation of Differences between Mapped and Measured Fluxes * (%)		Largest Single Difference between Mapped and Measured Fluxes* (%)		Largest Difference in Zone Flux between (S) and (E) (MW)	Number of Zone-Flux Differences Greater than 1 MW
	(S)	(E)	(S)	(E)		
0	2.81	2.81	7.3	7.3	0.00	0
1	2.81	2.81	7.4	7.4	0.00	0
5	2.83	2.82	7.2	7.4	0.02	0
7	2.92	2.88	8.6	7.6	0.81	0
10	2.96	2.92	8.6	7.6	1.43	1
15	3.51	2.81	10.2	6.5	3.33	10
18	3.09	2.70	6.9	5.9	2.42	5
20	3.08	2.66	7.5	5.9	2.36	7
22	3.52	2.73	10.4	6.1	4.14	10
25	3.15	2.67	7.7	5.9	2.92	8
30	3.16	2.50	11.1	5.7	6.84	9

* Computed over "good" (non-irrational) detectors