AN EMPIRICAL TWO-FLUID MODEL FOR CRITICAL FLOWS

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ABSTRACT

The object of this paper is to propose an empirical critical flow model, based on the two-fluid results, to account for phase slip, thermal non-equilibrium and piping geometric effects. The main emphasis is on the derivation of the model which includes the major relevant physical phenomena, and it can be used to make predictions suitable for comparison with experimental data. The critical flow rate is formulated in a simple form: non-homogeneous and non-equilibrium factors time the homogeneous equilibrium critical flow rate.

INTRODUCTION

Prediction of the break discharge flow rate is important in the safety analyses of nuclear reactors. Over the past two decades, there have been many theoretical and experimental studies of the critical flow. In general, there are two theoretical approaches: empirical (for example, References 1-3) and mechanistic (References 4-6) models. In the first approach, one or more empirical parameters are introduced and fitted to experimental data. As a result, simplified correlations or tables can be reproduced and applied to the reactor thermalhydraulic codes. This approach has been widely applied in engineering applications. The most used critical flow models in nuclear industry are the Moody (Reference 2) and the Henry and Fauskey (Reference 3) models. In the second approach, the non-homogeneous and non-equilibrium effects are introduced through the two-fluid or simplified two-fluid equations. The unsteady or steady state equations are then integrated along the pipe direction until the choking condition is satisfied.

The problems related to critical flows have been reviewed by many authors (for example, References 7-10). It has generally been found that the critical flow rate predicted by the homogenous equilibrium (EVET) model are considerably lower than those obtained experimentally. The inclusion of phase slip has received considerable attention. The equilibrium models which incorporate high velocity ratio (Reference 11) show good agreement with experimental data. However, the velocity ratios predicted by these models are much greater than those observed experimentally. Experiments for discharges from short pipes or nozzles indicated substantial liquid superheat. Since then, the phenomenon of thermal non-equilibrium has received considerable attention (References 3, 5, 6, 12).

In the critical flow formulation, the following theoretical aspects must be considered:

(1) Formulation of the critical flow as a function of local or exit conditions. This is the essential part of the critical flow theory. In the classical critical flow theory, the assumption of a maximum flow rate with respect to the throat pressure is applied. In the two-fluid theory, a stationary rarefaction wave is utilized.

(2) Relations between the throat and the upstream or stagnation conditions. This is the important part for modelling applications. Different assumptions which lead to different physical meanings have been made in the existing empirical models. For example, an isentropic assumption was used in the Moody model (Reference 2) and a constant quality was assumed in the Henry and Fauske model (Reference 3).

The object of this paper is to propose an empirical critical flow model, based on the two-fluid results, to account for phase slip

(UV), thermal non-equilibrium (UT) and pipe geometry effects. The main emphasis is on the derivation of the model which includes the major relevant physical phenomena, and it can be used to make predictions suitable for comparison with experimental data.

The leak before break concept is being discussed in the Canadian nuclear industry. It is important to clarify the leak flow rate since it relates to the methods of leak detection and to confirm the leak location. The discharge flow rate from very narrow and short slits may require further consideration in the analytical model, especially the thermal non-equilibrium effect.

THEORETICAL CONSIDERATIONS

A rupture of a pipe will generate a rarefaction wave which travels downstream at sonic velocity relative to the flow. When the flow velocity at break becomes equal and opposite to the sonic velocity, the rarefaction wave becomes stationary and the flow chokes or becomes critical. The critical flow rate is then determinated by the upstream local conditions, but not by the downstream conditions. The critical flow rate is computed from

$$W_{critical} = C_d AG = C_d A\rho a \tag{1}$$

where A is the break area, C_d is the discharge coefficient which depends on the break type and break orientation, W is the flow rate, G is the mass flux, ρ is the exit fluid density and a is the sound speed at the exit plane which depends on the model assumed. It will be shown that Equation (1) can be derived from the classical critical flow theory, but it is only an approximate result from the two-fluid theory.

Before the results based on the two-fluid model are presented, the classical critical flow theory is reviewed in order to highlight differences to the two-fluid model developed in this report.

Classical Theory

In the classical critical flow theory, the mixture momentum equation at a steady state along the flow direction becomes:

$$\frac{\partial}{\partial s}\left[G^2\left(\frac{\tilde{x}_g^2}{\alpha_g\rho_g} + \frac{\tilde{x}_f^2}{\alpha_f\rho_f}\right)\right] + \frac{\partial p}{\partial s} = 0 \tag{2}$$

where the subscripts g,f denote vapour and liquid, respectively, s is the flow direction, p is the pressure, G is mass flux (G =

W/A), α_k is the void fraction of phase k, ρ_k is the phase density and $\bar{x}_k$ is the flow quality which is defined by

$$\bar{x}_k = \frac{\alpha_k \ \rho_k \ v_k}{G} \tag{3}$$

where v_k is the phase velocity. The mass flux and the mixture density are defined by

$$G = \alpha_g \ \rho_g \ v_g + \alpha_f \ \rho_f \ v_f$$
$$\rho = \alpha_g \ \rho_g + \alpha_f \ \rho_f \tag{4}$$

At critical flows, it is assumed that the mass flow rate exhibits a maximum with respect to the pressure at the exit plane:

$$\frac{dG}{dp} = 0 \tag{5}$$

As a result, the mixture momentum equation yields

$$G_{critical} = \rho \ a \tag{6}$$

where the sound speed at the throat is defined by

$$a = \left[\rho \sqrt{- \frac{dv_m}{dp}} \right]^{-1} \tag{7}$$

and the volumetric density is given by

$$v_m = \frac{\bar{x}_g^2}{\alpha_g \ \rho_g} + \frac{\bar{x}_f^2}{\alpha_f \ \rho_f} \tag{8}$$

Define the velocity or slip ratio k by

$$k = \frac{v_g}{v_f} \tag{9}$$

and using the general relation between void fraction and flow quality,

$$\frac{\alpha_f}{\alpha_g} = \frac{\tilde{x}_f}{\tilde{x}_g} k \frac{\rho_g}{\rho_f} \tag{10}$$

the volumetric density v_m becomes

$$v_m = \left(\tilde{x}_g + \frac{\tilde{x}_f}{k} \right) \left(\frac{\tilde{x}_g}{\rho_g} + \frac{k \, \tilde{x}_f}{\rho_f} \right) \tag{11}$$

Therefore, the sound speed of the classical critical flow theory is a function of $\tilde{x}_k$, ρ_k and k at the throat. The pressure dependence of the slip ratio k is usually neglected in the classical critical flow models. However, it is an important term in the two-fluid theory.

The relations between the flow quality $\tilde{x}_k$ and the static (or thermodynamic) quality x_k are given by

$$\tilde{x}_g = x_g + \theta \quad ,$$
$$\tilde{x}_f = x_f - \theta \tag{12}$$

where

$$\theta = \frac{x_g \, x_f (k - 1)}{x_g \, k + x_f} \tag{13}$$
$$x_k = \alpha_k \, \rho_k / \rho$$

Instead of the flow quality, the thermodynamic quality x_k is used in the two-fluid model.

In an earlier isentropic thermal equilibrium theory, Fauske (Reference 11) used a slip ratio proportional to the square root of the phase density ratio, which minimizes the momentum flux at the exit. Moody (Reference 2) assumed a slip ratio proportional to the cubic root of the phase density ratio, which was shown to maximize the isentropic flow rate with respect to k.

Several slip correlations have been proposed for the subcritical flow. For example, the Bankoff slip correlation (Reference 13) has considered the pressure effect, and the Bryce slip correlation (Reference 14) has included the pressure and mass flow effects. None of those correlations are applicable to the critical flow condition.

A number of theories have explicitly allowed for the possibility of thermal non-equilibrium between the two phases due to finite vapour generation. Henry and Fauske (Reference 3) have developed a simple non-equilibrium model for the critical flow rate. They have described the non-equilibrium behaviour in terms of an empirical parameter which relates the actual change in quality with pressure to the rate of change occurring under homogeneous isentropic equilibrium conditions.

Two-Fluid Theory

In the two-fluid model, the non-homogeneous and non-equilibrium effects are introduced through the governing equations. In a critical flow, the possible flow regimes encountered are bubbly, churn turbulent and highly dispersed mist flows. In the phase momentum equation for those flow regimes, the virtual mass force is given by

$$f_{vm} = -C_{vm}\alpha_g\rho_f\left[\frac{\partial}{\partial t}(v_g - v_f) + v_g\frac{\partial v\text{-}g}{\partial s} - v_f\frac{\partial v_f}{\partial s}\right] \tag{14}$$

where C_{vm} is the virtual mass coefficient resulting from the streamline deflection around the interface.

From the phase momentum equations, the velocity ratio during a fast transient is given by

$$k = \frac{\rho_f + C_{vm}\,\rho_f/\alpha_f}{\rho_g + C_{vm}\,\rho_f/\alpha_f} \tag{15}$$

which defines the relationship between the velocity ratio and the virtual mass coefficient. For a steady state flow, the velocity ratio becomes

$$k_{ss} = \sqrt{k} \tag{16}$$

For a well separated (smooth stratified or annular) flow with zero virtual mass coefficient, the phase velocity ratio in a steady state is reduced to that of the Fauske slip (Reference 11). For a perfect mixed (homogeneous) flow with an infinite virtual mass coefficient, the phase velocity ratio is one.

In the thermal non-equilibrium model, the vapour generation rate consists of three components: flashing pressure transient, interface heat transfer and wall heat transfer terms. The interface and wall heat transfer terms contain no time derivative and, therefore, it has no effect on the characteristic analysis of the governing equations. The flashing mass transfer rate due to the pressure transient is given by

$$\Delta m_g = -\sum_k q_{kp}/(h_{gs} - h_{fs}) \quad , \quad k = g,f \tag{17}$$

where h_{ks} is the specific enthalpy of saturated phase k and

$$q_{kp} = \beta \left[\alpha_k \rho_k \frac{dh_{ks}}{dp} - \alpha_k\right]\left[\frac{\partial p}{\partial t} + v_k \frac{\partial p}{\partial s}\right] \tag{18}$$

where β is the boiling parameter indicating the degree of mass transfer between two phases due to fast depressurization process (β = 0 for a frozen model and 1 for the thermal equilibrium model).

From the characteristic analysis of the two-fluid equations (Reference 15), the flow chokes when the mixture velocity v is equal to the sound speed a, or

$$v = a \quad , \tag{19}$$

where the mixture velocity at the throat is given approximately by

$$\begin{aligned} v = \{ & \alpha_f \rho_g v_f + 2\rho_g \gamma_g v_f + 2\rho_f \gamma_f v_g \\ & + \gamma_g \gamma_f (\alpha_g \rho)g\, v_g + \alpha_f \rho_f v_f)/(\alpha)g\, \alpha_f) \\ & + (\rho_f - \rho_g)\, [\alpha_g \gamma_g v_g - \alpha_f \gamma_f v_f - \gamma_g \gamma_f (v_f - v_g)]\} \\ & / \, [(1 + \gamma_g + \gamma_f)\, (\alpha_f \rho_g + \alpha_g \rho_f \rho_g \gamma_g + \rho_f \gamma_f)] \end{aligned} \tag{20}$$

and γ_k is defined by

$$\gamma_k = C_{vm}\, \alpha_g \rho_f \,/\, (\alpha_k \rho_k) \tag{21}$$

For a highly dispersed flow with a large C_{vm}, the mixture velocity becomes

$$v = (\alpha_g \rho_g v_g + \alpha_f \rho_f v_f) \,/\, \rho \tag{22}$$

Consequently, the critical flow rate can be calculated approximately from Equation (6). The sound speed at the throat can be written approximately by

$$a = \left[\frac{\beta}{a_{UVET}^2} + \frac{1 - \beta}{a_{UVUT}^2} \right]^{-\frac{1}{2}} \tag{23}$$

where the a_{UVET} is the non-homogeneous thermal equilibrium sound speed and a_{UVUT} is the non-homogeneous thermal-frozen sound speed.

Thermal Equilibrium

In the thermal equilibrium model, the following equations of state are applied:

$$\begin{aligned} \rho_k &= \rho_{ks}(p) \\ u_k &= u_{ks}(p) \end{aligned} \tag{24}$$

where the subscript s denotes saturation value and u_k is the internal energy of phase k.

The thermal equilibrium sound speed obtained from the characteristic equation can be written in the form:

$$a_{UVET} = F \left[\rho^2 \left(\frac{x_g}{\rho_g^2 \, a_g^2(ET)} + \frac{x_f}{\rho_f^2 \, a_f^2(ET)} \right) \right]^{-\frac{1}{2}} \tag{25}$$

where F is a factor containing the non-homogeneous effect which is given by

$$F = \left[\rho \left(\frac{\alpha_g}{\rho_g} + \frac{\alpha_f}{\rho_f} \right) \frac{1 + \dfrac{C_{vm}\,\rho_f}{\alpha_g\,\alpha_f\,\rho_f + \alpha_f^2\,\rho_g}}{1 + \dfrac{C_{vm}\,\rho}{\alpha_f\,\rho_g}} \right]^{\frac{1}{2}} \qquad (26)$$

and the phase equilibrium sound speed $a_k(ET)$ is defined by

$$a_k(ET) = \left[\frac{1 + \dfrac{p\left(\dfrac{1}{\rho_g} - \dfrac{1}{\rho_f}\right)}{u_{gs} - u_{fs}}}{\dfrac{d\rho_{ks}}{dp} + \dfrac{\rho_k^2\left(\dfrac{1}{\rho_g} - \dfrac{1}{\rho_f}\right)}{u_{gs} - u_{fs}} \dfrac{du_{ks}}{dp}} \right]^{-\frac{1}{2}} \qquad (27)$$

The equilibrium sound speed falls continuously as the void fraction is reduced from one to zero. The homogeneous equilibrium sound speed is obtained from Equation (25) with F = 1, which corresponds to the case with an infinite virtual mass coefficient.

Thermal Frozen

The frozen model assumes no interfacial heat transfer between two phases. The thermal frozen sound speed can be written approximately by

$$a_{UVUT} = F \left[\rho^2 \left(\frac{x_g}{\rho_g^2\, a_g^2(UT)} + \frac{x_f}{\rho_f^2\, a_f^2\,(UT)} \right) \right]^{-\frac{1}{2}} \qquad (28)$$

where F is given by Equation (26). The sound speed for phase k is given by

$$a_k(UT) = \left[\frac{1 - \dfrac{p}{\rho_k^2} \left(\dfrac{\partial \rho_k}{\partial u_k} \right)_p}{\left(\dfrac{\partial \rho_k}{\partial p} \right)_{u_k}} \right]^{\frac{1}{2}} \qquad (29)$$

where the following equation of state is applied:

$$\rho_k = \rho_k \, (p, u_k) \tag{30}$$

The sound speed of a frozen model falls from that of vapour at a void fraction of one until a particular void fraction is reached. Then it reaches the liquid sound speed as the void fraction is decreased to zero. The homogeneous frozen model is obtained from Equation (28) with F = 1. For the cases with low qualities, a frozen model has been applied by Henry and Fauske (Reference 16).

Critical Frozen

It is known that the liquid sound speed is much larger than the critical liquid velocity obtained from the Bernoulli equation for a subcooled critical flow. For a subcooled flow at throat, the critical flow rate is usually calculated from:

$$G = [2\rho_f \, (p - p_\infty)]^{\frac{1}{2}} \tag{31}$$

where p_∞ is the downstream pressure.

In the critical frozen model, the liquid phase at the front of expansion wave undergo a minimum mass transfer which limits the expansion wave velocity. The following formula is applied for the density derivative with respect to pressure for the liquid phase in Equation (29):

$$\frac{\partial \rho_f}{\partial p} = \frac{1}{2} \frac{\rho_f}{p} \tag{32}$$

This formula is derived from Equation (31) when $p_\infty = 0$. The critical frozen sound speed has the same physical trend as the equilibrium sound speed for low and intermediate pressures. Both fall from the speed of sound in the vapour phase at a void fraction of one until the void fraction is decreased to zero. However, they have different physical meanings in their formulations.

In this study, the critical frozen model is used instead of the thermal frozen model in the calculation of the system sound speed of a critical flow.

In summary, the critical flow rate based on the two-fluid model can be expressed by

$$G_{critical} = \frac{F\, G_{EVET}}{\sqrt{\beta + \frac{1 - \beta}{C^2}}} \tag{33}$$

from Equation (23), where C is the ratio of the homogeneous critical frozen sound speed to the homogeneous equilibrium sound speed:

$$C = a_{EVUT} \,/\, a_{EVET} \tag{34}$$

and the non-homogeneous factor F is a function of void fraction and virtual mass coefficient. The parameters that affect the sound speed are the virtual mass coefficient C_{vm} and the boiling parameter β.

SIMPLIFIED TWO-FLUID EQUATIONS

In the previous section, the critical flow rate as a function of local throat conditions (p, x, u_g, u_f) or (p, x, h_g, h_f) has been formulated. The important physical values that affect the sound speed at throat are pressure and quality. The local throat conditions as functions of upstream entrance or stagnation conditions can be obtained from the simplified steady state two-fluid equations.

Steady State Equations

From the phase momentum equation, the change of the phase velocity is given by

$$\rho_k^{\bullet}\, d\left(\frac{v_k^2}{2}\right) + dp + f_w = 0 \tag{35}$$

where

$$\rho_g^{\bullet} = \frac{\rho_g\, \rho_f\, (1 + \gamma_g + \gamma_f)}{\gamma_g\, \rho_g + (1 + \gamma_f)\, \rho_f}$$

$$\rho_f^{\bullet} = \frac{\rho_g\, \rho_f\, (1 + \gamma_g + \gamma_f)}{\gamma_f\, \rho_f + (1 + \gamma_g)\, \rho_g}$$

and f_w is the pressure drop due to wall friction which is calculated from

$$f_w = \frac{G^2}{2\rho}\frac{fL}{D} \tag{36}$$

where ρ is the upstream mixture density, D is the diameter, f is the Darcy friction factor which is four times the Fanning friction factor, and L is the discharge length.

From the phase energy equations, the change of phase specific enthalpy is given approximately by

$$dh_k \approx \left[\beta \frac{dh_{ks}}{dp} + \frac{1-\beta}{\rho_k} \right] dp \tag{37}$$

From the mass conservation of vapour phase, the change of the flow quality is given by

$$d\tilde{x}_g = \Delta m_g / G \tag{38}$$

where the vapour generation rate is given by

$$\Delta m_g = -\frac{\beta G}{h_{gs} - h_{fs}} \left[\left(\frac{dh_{gs}}{dp} - \frac{1}{\rho_g} \right) \tilde{x}_g + \left(\frac{dh_{fs}}{dp} - \frac{1}{\rho_f} \right) \tilde{x}_f \right] dp \tag{39}$$

From Equations (37) - (38), the change of the flow enthalpy of the mixture satisfies

$$d\tilde{h} = \left(\frac{\tilde{x}_g}{\rho_g} + \frac{\tilde{x}_f}{\rho_f} \right) dp \tag{40}$$

where

$$\tilde{h} = \tilde{x}_g h_g + \tilde{x}_f h_f$$

Numerical Procedure

The numerical procedures to obtain the critical flow rate and the critical pressure ratio are as follows:

(1) Assume a pressure drop dp.
(2) Assume a flow rate and find the throat pressure.
(3) Estimate the pressure drop due to the wall friction.
(4) Calculate the phase velocities.
(5) Calculate the phase specific enthalpies and the flow quality.
(6) Find the phase internal energies and the static quality.

(7) With known (p, x, u_g, u_f), call the equations of state to find other physical properties.
(8) Calculate the sound speed, a.
(9) Calculate the mixture velocity, v.
(10) Calculate the flow rate and go to step (2) until the variations of the sound speed and the flow rate are small enough.
(11) If the mixture velocity v is smaller than the sound speed a, guess a new pressure drop and go to step (1) until the condition v = a is satisfied. If the throat pressure is less than the downstream pressure, the flow is subcritical.
(12) Calculate the critical flow rate and the critical pressure ratio (p(throat) / p_o).

DISCUSSIONS OF MODEL

Non-Homogeneous Parameter

In the critical flow, the possible flow regimes are either bubbly, churn turbulent or highly dispersed flows. Different forms of the virtual mass coefficients for subcritical flows have been suggested in the literature. The virtual mass coefficient is 0.5 for isolated spheres in a potential flow field. However, it has been shown (Reference 17) that the virtual mass coefficient becomes larger than 0.5 when closely spaced spheres interact through their potential flow fields. Experimental evidence is cited to increase values of the virtual mass coefficient up to about 1.5 in bubbly flow. Most correlations show that the virtual mass coefficient increases as the void fraction increases. However, prints (Reference 17) has shown that the virtual mass coefficient rapidly decreases with void fraction from experiment. Also it has been shown from critical flow experiments (Reference 18) that the system pressure has strong influence on the velocity ratio.

In this work, the following simple form is suggested:

$$C_{vm} = 1 - \alpha_g^n \tag{42}$$

where the parameter n is expected to be a function of pressure. The following value of n is assumed:

$$n = 0.5\ p \tag{43}$$

where p is the pressure measured in MPa.

Figure 1 shows the comparison of the thermal equilibrium, thermal frozen and critical frozen sound speeds of a homogeneous model at p = 1 MPa for heavy water. The corresponding non-homogeneous factor is shown in Figure 2 to illustrate the effect of slip ratio on the sound speed. It shows that the slip effect increases as the void fraction increases. It is noted that the slip effect vanishes when the void fraction is either zero or one. This contradicts with the result found by Baum and Horn (Reference 19). They showed that the slip effect still exits even when the void fraction is zero. On the other hand, Moody (Reference 20) has shown the effect of slip ratio in his critical flow model. His critical flow rates with a slip value of unity are generally higher than those computed using his maximum stable slip ratio. This also contradicts with the result shown in Figure 2 that the homogeneous flow (k = 1) leads to critical discharge rates below those predicted by the slip model. This discrepancy may result from the difference between the classical and two-fluid critical flow theories.

The validity of the slip model is best demonstrated by comparisons with experimental data on the slip ratios. The prediction and test results of Klingebiel (Reference 18) under a critical flow condition with different pressures is shown in Figure 3.

Non-Equilibrium Parameter

Barclay et al (Reference 22) have measured the sound speed for rarefaction waves in a boiling water-steam mixture at p = 0.1 MPa. The measured sound speed is much lower than that predicted by a thermal frozen model and higher than the thermal equilibrium value. Figure 4 shows the comparison of theoretical and experimental sound speeds. It shows that the critical frozen model agrees very well with the experimental data.

Based on experimental data, Henry and Fauske (Reference 3) have suggested the non-equilibrium parameter as a function of equilibrium quality for the low quality case. It should be noted that their correlations were fitted to the experimental data based on a homogeneous model.

The important parameters that affect the thermal condition at the throat are the flow quality and the absolute length L of the tube (or the passing time of fluid through the nozzle or tube). From the bubble growth theory of a bubble in a uniformly superheated liquid, Pinto and Davis (Reference 23) have shown that the bubble radius is a function of the square root of time. This square root of time law has been verified experimentally. In the present model, the boiling parameter is assumed proportional to the square root of the discharge length L. Therefore, the boiling parameter β for saturated and two-phase discharges is written as

$$\beta = \sqrt{\frac{L}{L_{eq}}} \qquad \textbf{(44A)}$$

where L_{eq} is the length required to establish a thermal equilibrium condition from the location of saturation line. From the experimental data, it is estimated that L_{eq} is about 300 mm.

For a subcooled discharge, the boiling parameter can be written as

$$\beta = \sqrt{(L - L_{sat}) / L_{eq}} \tag{43B}$$

where L_{sat} is the length from the upstream subcooled location to the point of saturated pressure.

If the upstream conditions are not available, the boiling parameter can be written in terms of the equilibrium quality at the throat. Similar to the form suggested by Henry and Fauske (Reference 3), the following simple form is suggested:

$$\beta = \sqrt{x_{eq}} \tag{44}$$

where x_{eq} is the equilibrium quality at the throat.

Figure 5 shows the ratio of homogeneous frozen and homogeneous equilibrium sound speeds, or C defined by Equation (34), as a function of pressure for heavy water at low void fraction. The variation of the sound speed ratio with the void fraction for heavy water is shown in Figure 6.

COMPARISONS WITH EXPERIMENTAL DATA

High Quality Critical Flow

At high quality flows, it is expected that the non-homogeneous effect is more important than the non-equilibrium effect on the critical flow. Figure 7 compares calculated critical flow rates with some available data given in Reference 1. It shows that the flow rates predicted by the homogeneous equilibrium model are considerably less than those obtained experimentally. The existence of slip between two phases is the main reason given to explain these discrepancies at high quality flow.

Low Quality Critical Flow

At low quality flows, it is expected that the non-equilibrium effect plays a much significant role than the non-homogeneous effect in the critical flow formulation. Figure 8 compares calculated critical flow rates with the experimental data of Henry and Fauske (Reference 24) at low qualities at p = 50 psia. The model generally exhibits fair agreement with the experimental data except at the very low qualities. Figure 9 illustrates the non-homogenous and non-equilibrium effects on the critical flow, where the experimental data were obtained from Reference 1.

Ratio of Length to Diameter

The effects of L/D on the critical flow are increasing the pressure drop due to wall friction and increasing the degree of thermal equilibrium or boiling parameters. Both effects tend to reduce the prediction of critical flow rate. When the value L/D is greater than 50, the pressure drop due to wall friction becomes important. It has been shown (Reference 25) from experiment that the critical mass flux to be approximately proportional to $(L/D)^{-0.7}$ when the L/D value is greater than 100. The non-equilibrium effects are empirically determined by the boiling parameter which characterize (1) the vapour generation rate and subsequent the non-equilibrium throat quality, and (2) the sound speed at throat.

Figure 10 shows the comparisons of critical discharge of saturated water through orifices, nozzles and pipes with experimental data taken from Reference 7 for different L/D values. Inspection of Figure 11 shows good agreement between predictions and measured saturated water critical flow rates. It indicates that the value L_{eq} = 300 mm chosen in Equation (43) is reasonable.

CONCLUSIONS

An empirical model based on the two-fluid theory has been developed to predict critical flow rates. The model approximates the non-homogeneous non-equilibrium processes by two empirical parameters which are determined experimentally. The critical flow rate is formulated in a simple form: non-homogeneous and non-equilibrium factors time the homogeneous equilibrium critical flow rate. The model exhibits good agreement with the experimental data for low and high quality flows.

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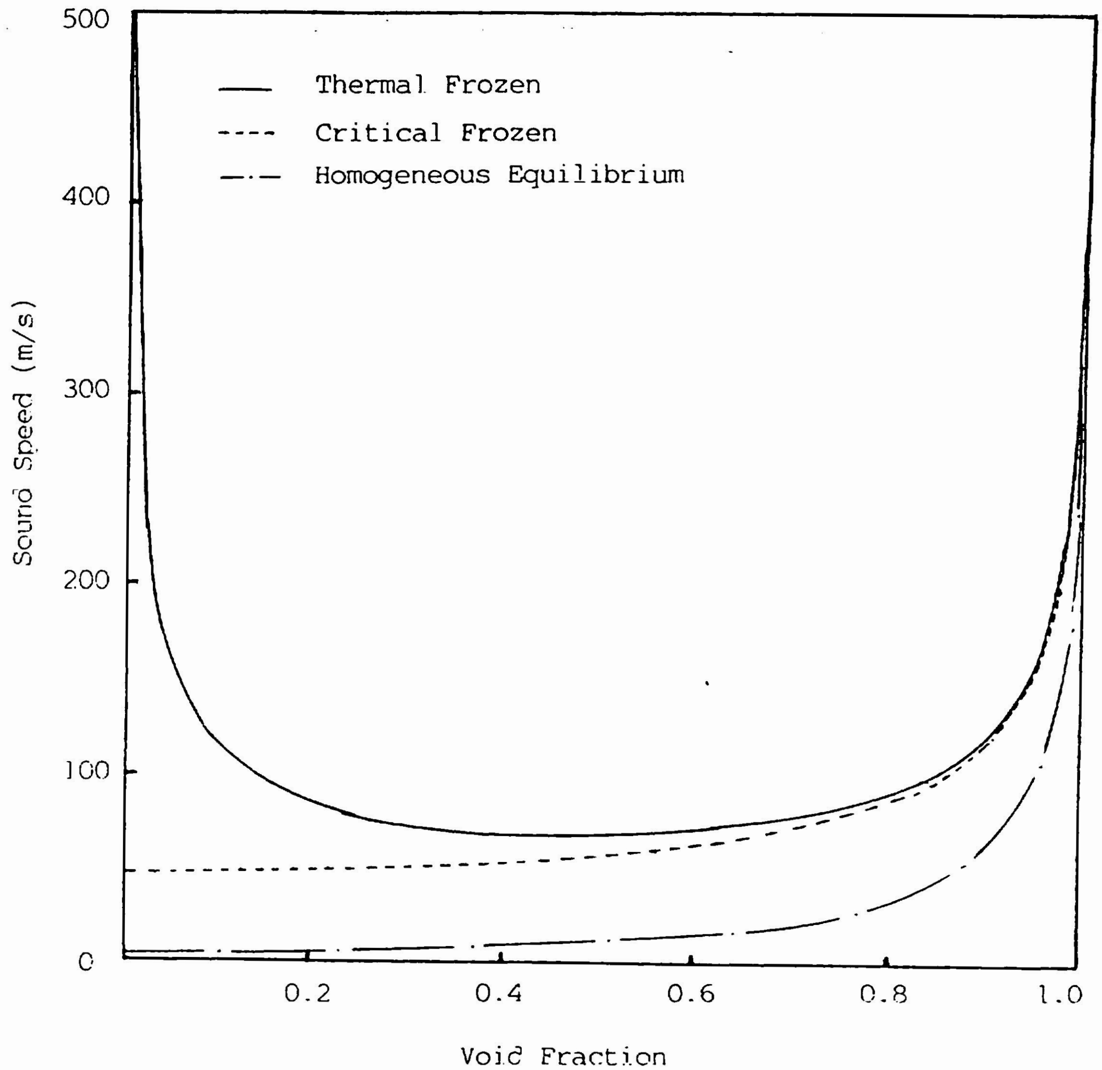

Figure 1. Comparison of the sound speeds of a homogeneous model at P = 1 MPa of heavy water

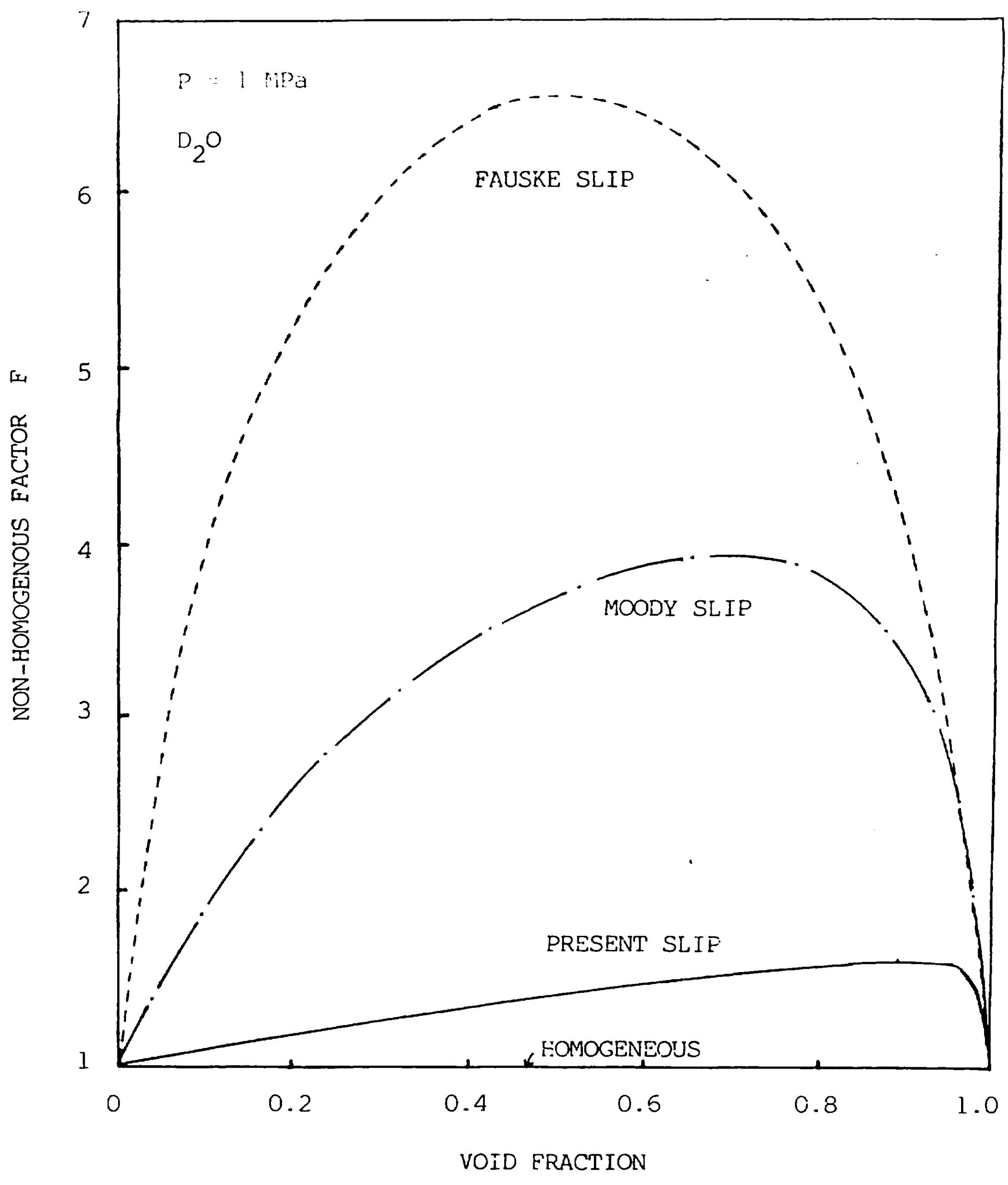

Figure 2. The non-homogeneous factor F as a function of void fraction at P = 1 MPa of heavy water

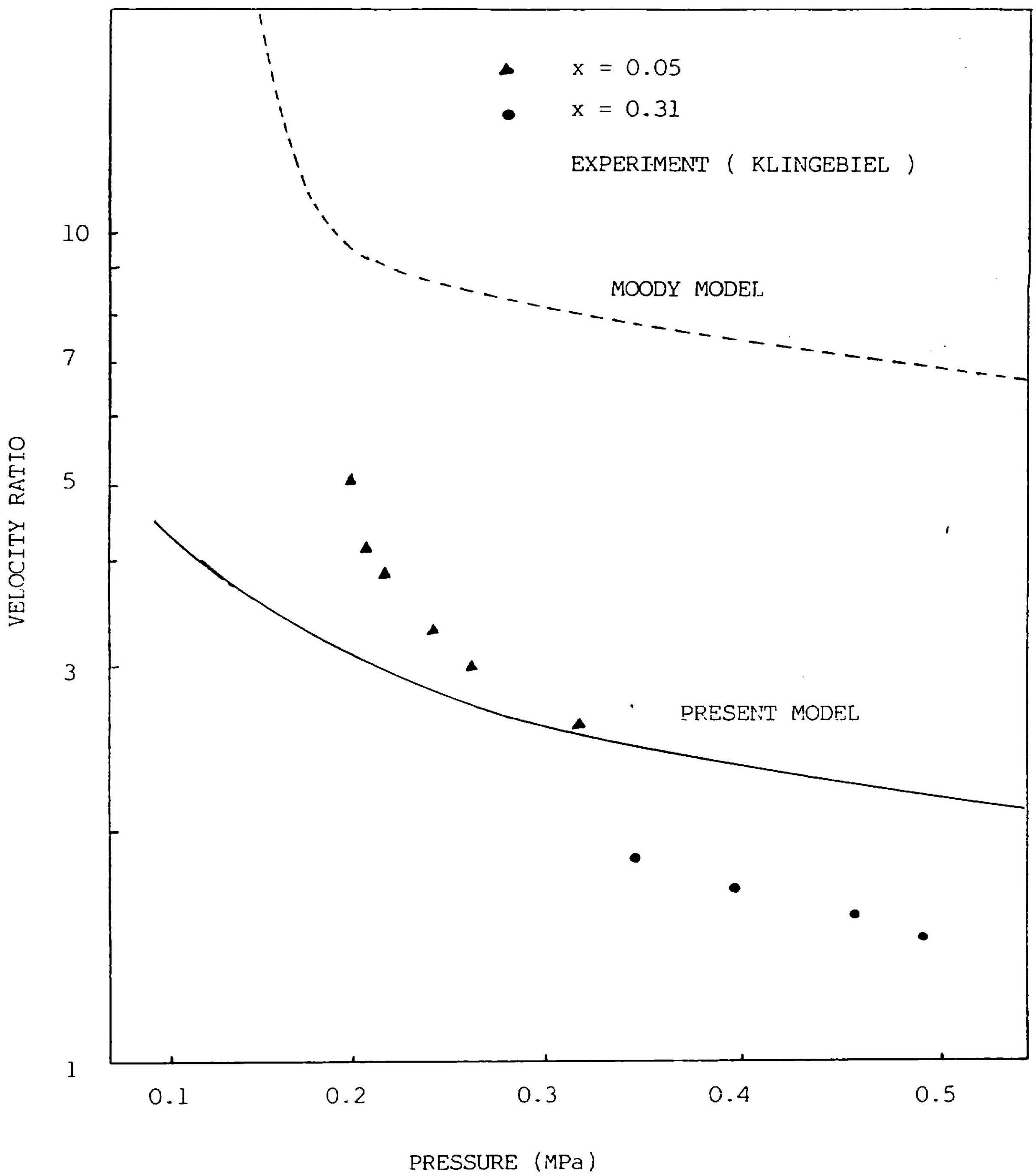

Figure 3. Comparison of theoretical and experimental slip ratios for steam water mixtures

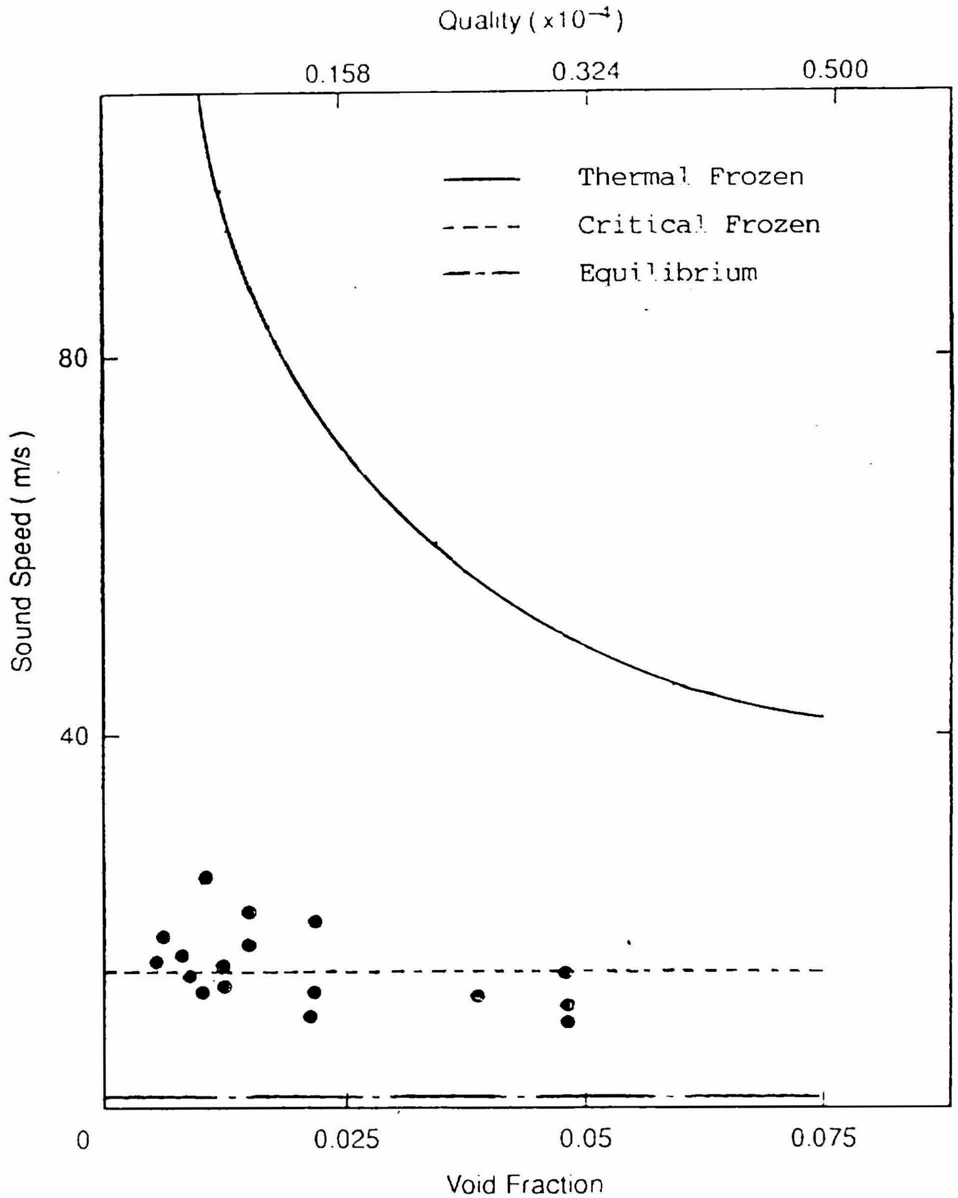

Figure 4. Comparison of sound speed with experimental data of Barclay et al. (p = 100 kPa)

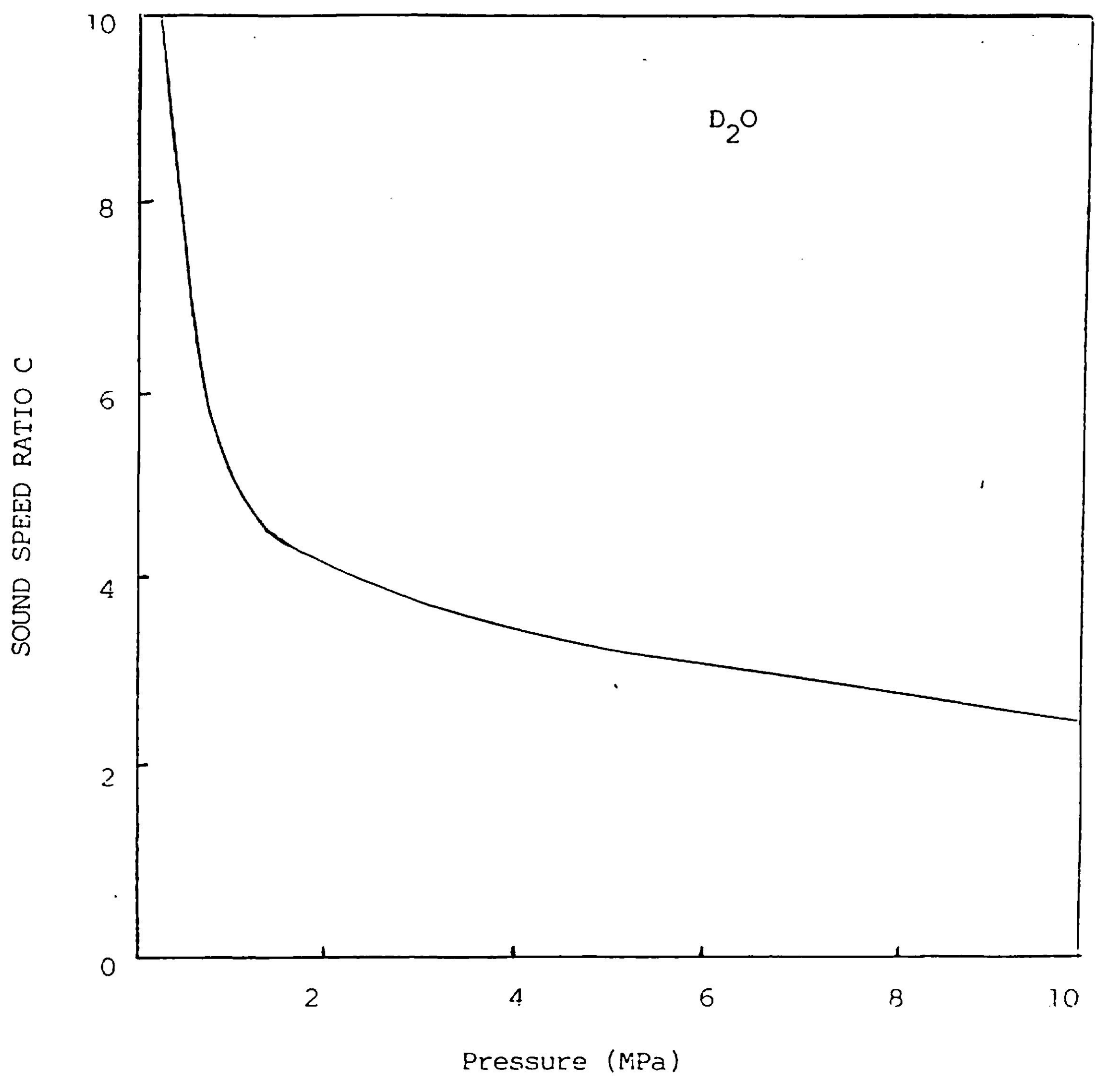

Figure 5. Ratio of the homogeneous frozen and homogeneous equilibrium sound speeds for heavy water at low void fraction

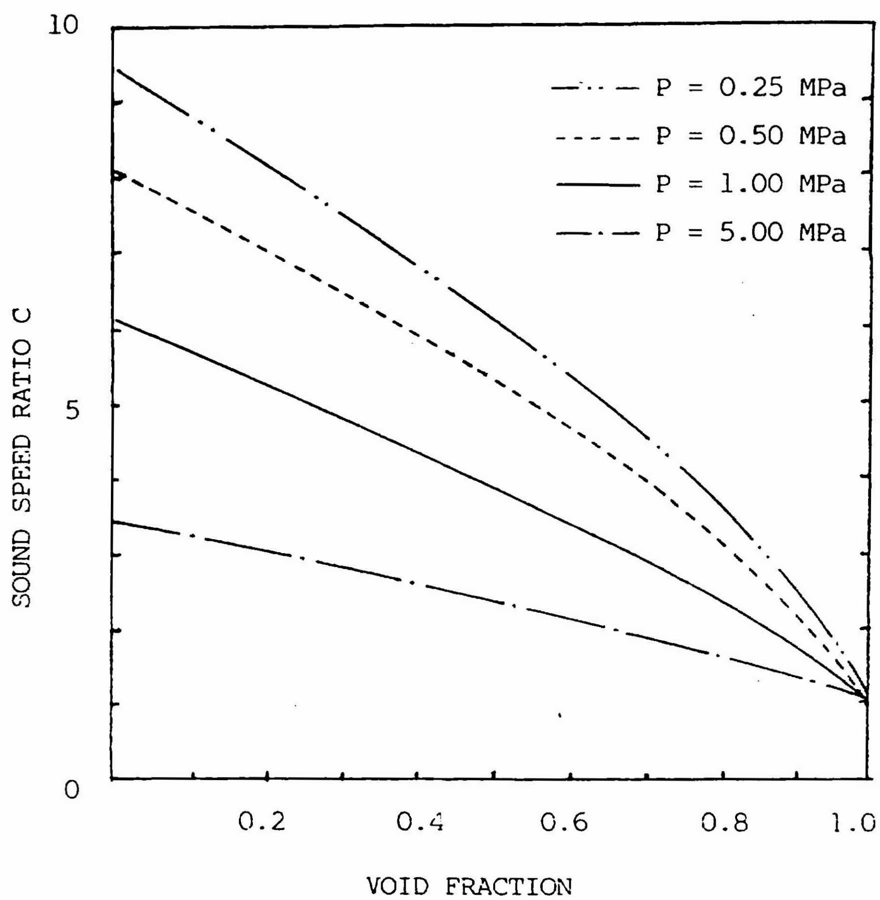

Figure 6. Ratio of the homogeneous frozen and homogeneous equilibrium sound speeds as a function of pressure

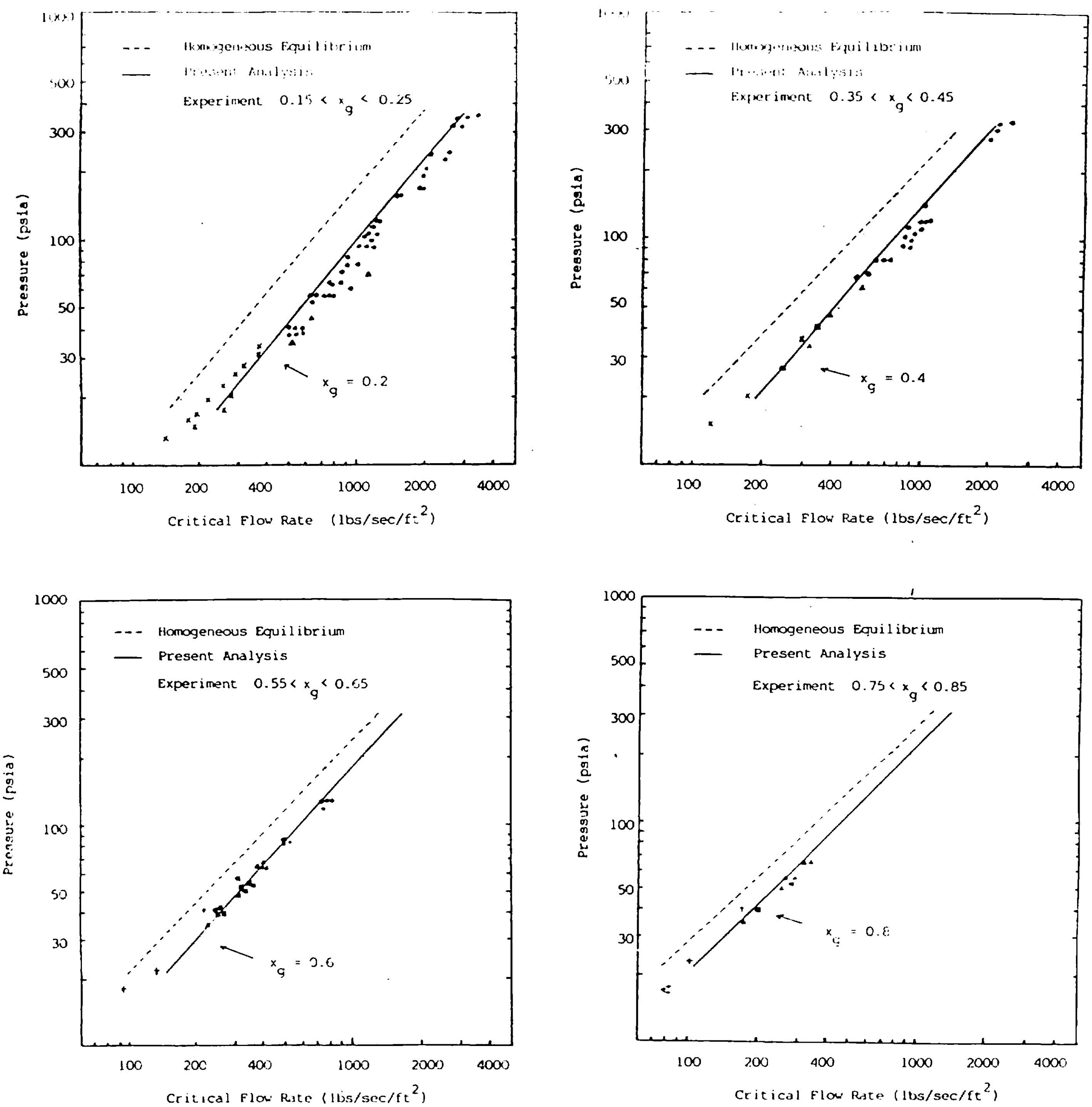

Figure 7. Critical flow rates

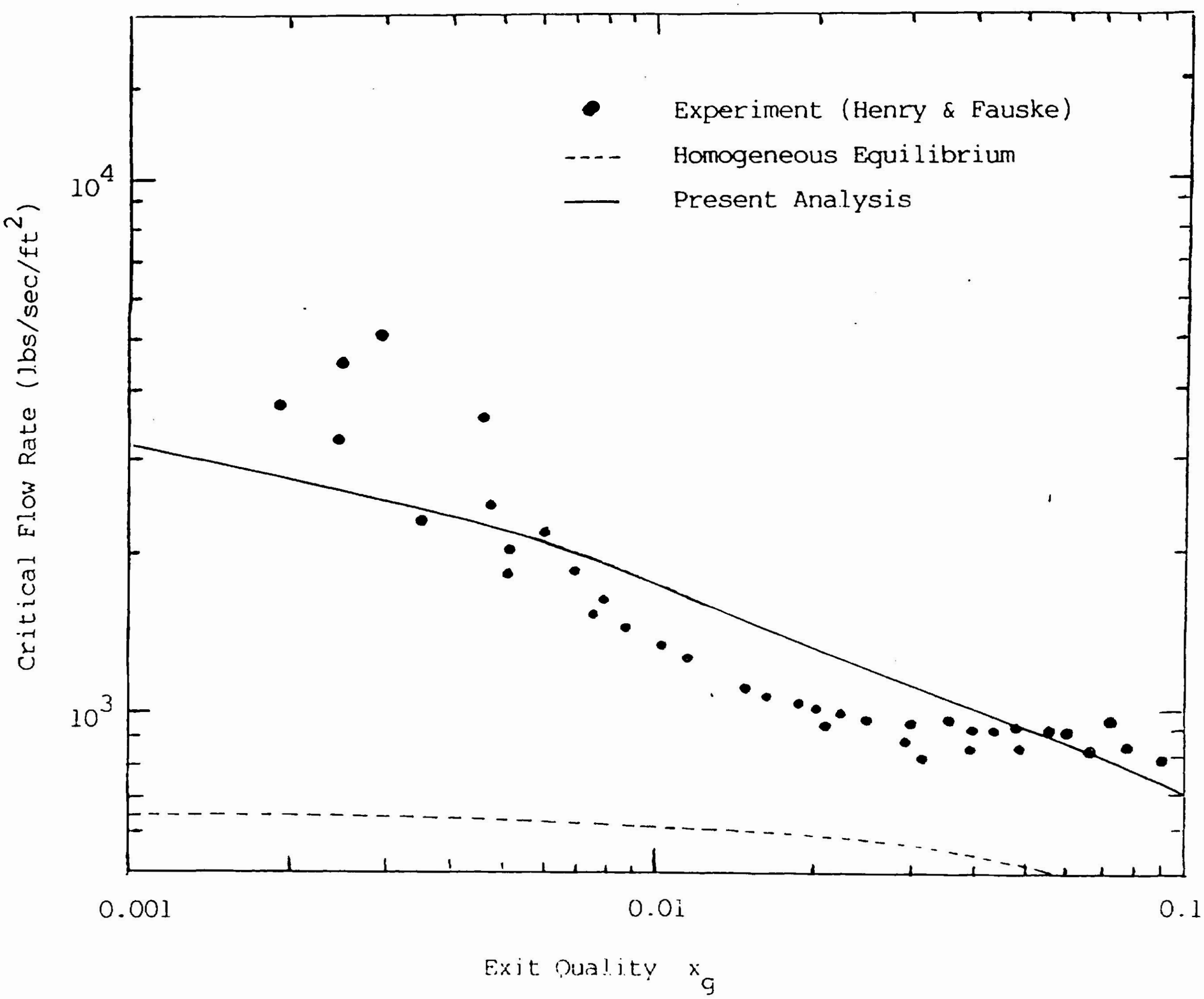

Figure 8. Comparison of predicted and experimental critical flow rates

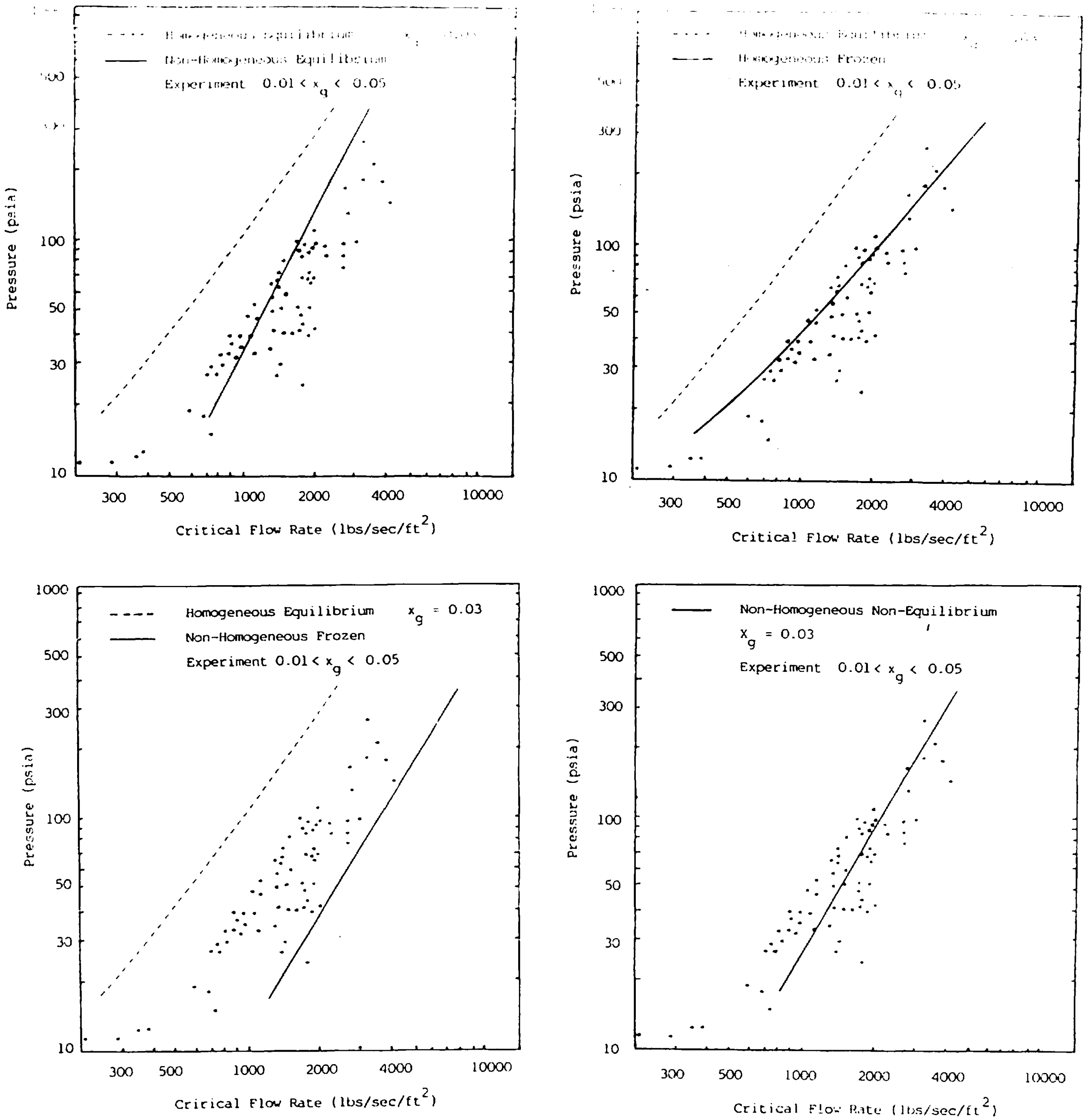

Figure 9. Critical flow rates

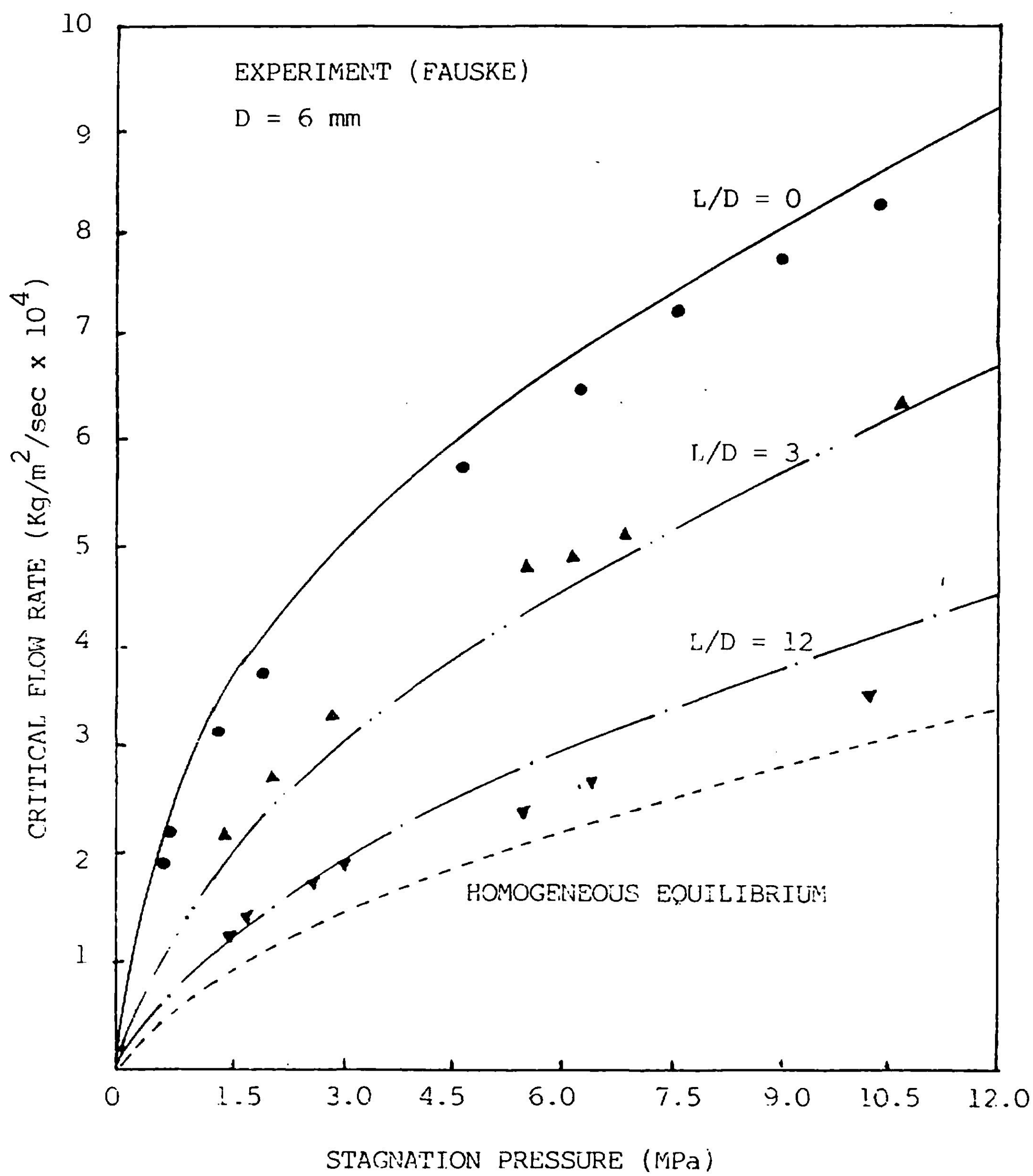

Figure 10. Critical flow rates as a function of L/D