THEORETICAL PREDICTION OF THE GARTER SPRING POSITIONS ALONG THE CHANNELS OF SOME CANDU REACTORS

CARLOS H.BORZI

GAID-CNEA and IFLYSIB-CONICET, Argentina

ABSTRACT

We propose a methodology for predicting the positions of the Garter Springs (GS) along the channels of those CANDU Reactors which have suffered displacements of the GS from their design positions. From the experimental location of the GS along some inspected channels we shall be able to predict the localization of the GS in the rest of the channels. We model the GS along a channel as beads along one abacus line (that is to say: hard cores in one dimension.) We use the Statistical Mechanics Theory to get the relationships between the mass distribution and the "external potential" which fully determines that distribution. That allows us to design the mentioned predicting procedure.

1.0 MOTIVATION AND STRATEGY

The motivation of this work is to build up a mathematical model on which we could study the problem of the displacement of the GS from their design positions along the channels of a CANDU reactor, where they act as separators between the pressure tube (PT) and the Calandria tube (CT). The model should be simple, in order to allow analytical treatment but it should contain all the features which are relevant to the subject problem. We will state a model which emphasizes the features we consider relevant and which is exactly solvable. Other relevant features, which we hope to learn from the readers, could be afterwards implemented on top of our simplifying assumptions. Our model will provide a way to interpolate information from some set of inspected channels in order to get information about the rest of them. The information required, that is the input to the implemented computer codes, is the set of positions and dispersions of the GS along as many channels as possible in a given reactor. The output of those codes will be the probability of finding a GS at a distance x along the axis of the rest of the channels. We shall see how good the model mimic the real system whenever we had a chance to analyze massive experimental data. We shall perform the alluded interpolation at the level of the causes whose effect is the displacement of the GS. We add together those causes in what we call the "external potential" u(x) which is related one to one to the "mass distribution" $\rho(x)$ in the non-uniform systems at equilibrium (x being a coordinate running along the axis of the channel from 0 to L.) Although the existence and uniqueness of the potential u(x) can be ensured in general, the chances to get a constructive formulation of the relationship between $\rho(x)$ and u(x) are zero for real non-uniform systems. Nevertheless, the formalism can be implemented on few model systems. We shall formulate one of those simplified models below. That model adapts particularly well to the physics of the GS and will allow us to explicitly give u(x) as a function of $\rho(x)$. Reciprocally, we shall get $\rho(x)$ and the correlations c(x1,x2,....) as functions of u(x). We shall discuss how to tune up the model on the basis of massive data from a given reactor as well as its further use on operating reactors where the data set is necessarily more restricted. We shall discuss in the appendix some aspects of the foundations of the model.

2.0 THE MODEL

We consider the GS as a set of hard cores in one dimension. That is, pictorially, one line of an abacus of length L (L = 600 cm) with 2 or 4 beads of length a (a = 0.6 cm). We are implicitly assuming that the GS are always perpendicular to the axis of the channel which is not realistic. Corrections due to tilting are easily estimated from a model already studied in the literature and result to be of the order of a/L = 0.001. We shall come back to this point in the appendix. The PT axis and the CT axis are pictured straight and coincident. Departures from that hypothesis will be related ultimately to the effect of gravity and are included in the "external potential". That potential will also include, through a convenient generalization of standard theories (see appendix) the effect of the vibrations of the Calandria, the friction of the GS with the tubes and all other interactions of the GS with the rest of the Universe. The system of hard rods in one dimension is one of the few exactly solvable models in the framework of the Equilibrium Statistical Mechanics Theory [1,2]. This system can be studied by itself in an infinite medium [3], confined to a finite length [4,5] and also when it interacts with an arbitrary external field [6]. We have adapted the theory developed in the references above (with few new contributions) through the equations summarized below:

2.1 The external potential

The external potential as a function of the distribution of mass in the finite length L, measured in units of kT, k being the Boltzmann constant and T being the absolute temperature:

$$\beta u(x) = -\ln z - \ln \rho(x) + \ln\left\{1 - \int_{x-a}^{x} \rho(\omega)d\omega\right\} - \int_{x}^{x+a} \frac{\rho(\eta)d\eta}{1 - \int_{\eta-a}^{\eta} \rho(\omega)d\omega} \qquad (1)$$

In eq.(1) the "activity" $z = \exp(-\beta\ \mu)$, μ being the "chemical potential" of the system in equilibrium, could be exactly evaluated [7] but it results more economical to fix it as an arbitrary constant because we will renormalize it anyway. We adopt the same criterion with respect to the "temperature" β. It is worth to remark here that u(x) in eq.(1) contains all the possible causes of the displacements of the GS from their design positions.

2.2 Probability of Presence of a GS

The probability of presence of a GS at a position x in the segment [0,L] is equivalent to the distributed density of mass in the Canonical Ensemble, which was obtained (through convenient Laplace transformations) in eq.(6) of Ref. [8]:

$$P(x) = \frac{\exp(-\beta u(x))}{Q_N(0,L)} \sum_{i=1}^{N} Q_{i-1}(0,x)\ Q_{N-i}(x,L) \qquad (2)$$

where:

$$Q_k(x,y) = \frac{1}{k!}\int_{x+a}^{x_1} \cdots \int_{x_k+a}^{y} \exp\left(-\beta \sum_{i=1}^{k} u(x_i)\right) dx_1 \cdots dx_k \qquad (3)$$

is the Canonical Partition Function of k hard cores in the segment [x,y] with the normalization $Q_0(x,y) = 1$.

2.3 The Conditioned Probability

The conditioned probability of finding a GS at position y when the first one was found at position x < y, was established in Ref.[4] and results to be:

$$P(y/x) = \frac{c(x,y)}{P(x)} \qquad (4)$$

where P(x) comes from eq.(2) and as we got in Ref.[8], with further corrections:

$$c(x,y) = \frac{\exp(-\beta(u(x)+u(y)))}{Q_N(0,L)} \sum_{j+k=1}^{N} Q_{j-1}(0,x) Q_k(x,y) Q_{N-j-k}(y,L) \qquad (5)$$

In these conditions we can design a procedure of interpolation which, as we announced above, will allow us to use the experimental knowledge of the positions and their dispersions in the inspected channels to predict the probability of presence of the GS and their correlations along the other channels. The procedure works as follows: Let us assume that we know the positions x^s_{ij} and their dispersions σ^s_{ij} along the axis of some channels. The superscript counts the GS and runs from 1 to 2 or from 1 to 4 depending on the reactor. The subscripts (i,j) serve to order the inspected channels on the section of the Calandria. We shall put indexes (i,j) to eqs.(1-5) when referring to those tubes. We identify a generic target channel with the pair (k,l). For each channel (i,j) we construct a continuous distribution of mass with two parameters per peak which get fixed after the experimental information. That information consists of the positions x^s_{ij} and the dispersions σ^s_{ij}. For the sake of simplicity we start proposing a sum of Gaussian distributions, conveniently renormalized:

$$\rho_{ij}(x) = \frac{1}{2\Pi N}\sum_{s=1}^{N}\frac{1}{\sigma^s_{ij}}\exp-\left(\frac{x-x^s_{ij}}{\sigma^s_{ij}}\right)^2 \qquad (6)$$

The generalization to more elaborated distributions of the type of the ones in Ref.[9,10] is immediate. From those distributions we evaluate the external potential (which is distributed in the space) along the axis of the inspected channels through eq.(1). Now we assume that the external potential is smooth enough as to be numerically interpolated within the Calandria. We shall interpolate on sections of the Calandria, that is on the indexes (i,j) for fixed values of x, and construct u(x) for the target channel (k,l) step by step. Once we know u(x) we use eqs.(2-5) to predict P(x) and P(y/x) for the target channel.

3.0 RESULTS

So far we only had at hand a set of randomly generated data [10] and a restricted set of experimental data [16,17]. The first mentioned set of data was generated on the basis of a proposed distribution which matches the first moments of a set of experimental data. Those generated data do not correspond to any real channel but in a global and indirect manner and are of little help in the process of judging the goodness of the model, since the whole procedure is designed for the investigation of specific individual channels. Nevertheless, the implementation of the process on that set of data allowed us to check that the model behaves as expected, from the Statistical Mechanics point of view, when it is fed with randomly generated numbers. The details can be seen in Ref [11].

The mentioned set of experimental data is presented in Table 1 at the end of the paper. In order to show how the model works, we take sets of 15 channels as input data and predict the positions, the dispersions and the relative error along the remaining one. We define the relative error at each channel as the average of the relative errors in the positions of the 4 garter springs in the channel. The results are presented in Table 2 also at the end of the paper. The global relative error of the whole calculation, what we estimate as the average relative error from the last column of Table 2, is: 22 %.

4.0 ASSESSMENT

The predictive value of our model must be analyzed after taking into account the following factors:

1) The subject data set is not big enough as to allow a proper tune up of the model. The normalizing constants and the proposed continuous distributions for the input densities have to be adjusted on the light of a massive set of data. Those data should be the positions and dispersions of the GS along some compact set of channels in some fully inspected Calandria. We insist in the necessity of analyzing such a data set in order to tune up the model and then to use it as a guide in the localization of the GS in operating reactors. The values in Table 2 are presented just as a demonstration of the potentiality of the model and, although they are not so bad, we are willing to recalculate them after we have a chance to actually do the mentioned tune up.

2) We are interpolating in two dimensions and the potential is defined in the space. We are currently working in the implementation of a three-dimensional interpolation process which could improve our predictions.

3) The average global error, which tries to summarize the goodness of the model, could be strongly affected by the size of the input data set, as we pointed out above. In order to give a realistic estimation of the practical relevance of our predictions we should be able to test them in other reactors and to study the dependence of the global error with the size and quality of the input data set.

On the light of the comments above, we have preferred no to pursue a detailed discussion of the results in Table 2. We could analyze correlations among the GS in the same channel, correlations among different channels as a way of establishing eventual symmetries, etc. We shall pursue that analysis after the model is properly tuned up as discussed above. Nevertheless, there is one feature of the results in Table 2 which is worth to be remarked: The relative error is bigger in those channels where the GS were found stuck together. That bigger error results to be natural since we are considering the GS as hard cores with a pure repulsive interaction. We shall include a "sticky interaction potential" among the GS, in future formulations of the model.

For the sake of completeness, we will briefly discuss the foundations of the model in the Appendix below.

5.0 APPENDIX

During the set up and the analysis of the model, several critics were pointed out by colleagues and friends. We shall discuss here the ones related to the foundations of the model.

The existence and Smoothness of the External Potential can be blindly ensured for conservative systems. The GS dissipate energy by friction with the tubes to which they are attached and so the subject system is not conservative. In addition, the presence of a random force (due to the vibrations of the Calandria) obscures much more the settlement of an equilibrium state and that is essential for the validity of the whole theory. Let us proceed by parts: First consider a system whose dynamics are dominated by friction or drag rather than by inertial or random forces. Let us assume that the system is subjected to an external potential u(x). The Newton's equations of motion read

$$\gamma\dot{x} = -\nabla u(x) - F(x) \qquad \text{(A1)}$$

where γ is the drag coefficient and we symbolize the time derivative with a dot and the gradient operator with ∇. This by itself will drive the system to the configuration of lowest potential energy. If the dissipation is countered by random forces, which while averaging zero will always pump energy into the system, the Newton's equations will look instead:

$$\gamma\dot{x} = -\nabla u(x) + A(t) \qquad \text{(A2)}$$

If we assume white noise, we would have:

$$\langle A(t)\rangle = 0 \ , \ \langle A(t)A(t')\rangle = \frac{2\gamma}{\beta}\,\delta(t-t') \qquad \text{(A3)}$$

And so eq.(A2) is a Langevin Equation. The zero correlation time assumption simply means here that the correlation time of the vibrations is short compared to the scale of non-random motion of the GS. With standard probability machinery we convert eq.(A2) into a Fokker-Planck Equation for the probability distribution of mass at x and t:

$$\gamma\frac{\partial\rho(x,t)}{\partial t} = -\nabla\cdot\left(\rho(x,t)\nabla u(x) + \frac{1}{\beta}\nabla\rho(x,t)\right) \qquad \text{(A4)}$$

Then the H-Theorem of Boltzmann shows how ρ(x,t) decays steadily to the stationary solution:

$$\rho(x) = K \exp(-\beta u(x)) \tag{A5}$$

We finally include the inertial terms and get:

$$\begin{aligned} M\dot{v} &= -\nabla u(x) - \gamma v + A(t) \\ v &= \dot{x} \end{aligned} \tag{A6}$$

By assuming white noise again and, after some computations, the interested reader could prove that ρ (x,v,t) settles down to a Thermal Ensemble and then the Equilibrium Statistical Mechanics applies [12].

The assumption of perpendicularity of the GS with respect to the axis of the channel is an idealization and corrections could be required. The case with tilting is already considered in the literature [13] and it results in the existence of an effective size of the cores. The influence on the variables of eqs.(1-5) is estimated to be of the order of a/L = 0.001.

The Ensemble Averages (which are essential to the Statistical Mechanics Theory) make real sense when dealing with systems with a big number of particles. If we think of a single channel we have, at most, 4 particles (the GS) and then we would fail at the time of changing Ensembles (through the Laplace Transformations mentioned in the text above) see Ref.[14]. To overcome that problem we shall think of a whole set of channels, which are replicas of the one of interest, and we shall perform the averages on the set of replicas. This is a trick first proposed by M.Kac [15]. In these conditions we get a selfconsistent model, even in the case with only 2 GS per channel.

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7.0 REFERENCES

[1] Percus, J.K. "Exactly Solvable Models of Classical Many Body Systems" in "Simple Models of Equilibrium and Non-Equilibrium Systems" J.L.Lebowitz editor. Elsevier Publisher, Amsterdam 1987.

[2] Baxter, R.J. "Exactly Solved Models in Statistical Mechanics" Academic Press, London (1982).

[3] Takahashi, H. "A Simple Method for Treating the Statistical Mechanics of One Dimensional Substances" Proc.Phys.Math. Japan 24:60 (1942).

[4] Salzburg, Z.W., Zwanzig, R.W. & Kirkwood, J.G. "Molecular Distribution Functions in a One Dimensional Fluid" J.Chem.Phys. 21:1098 (1953).

[5] Leff, H.S. & Coopersmith, M.H. "Translational Invariance Properties of a Finite One Dimensional Hard Core Fluid" J.Math.Phys. 8(2):306 (1967).

[6] Percus, J.K. "Equilibrium State of a Classical Fluid of Hard Rods in an External Field" J.Stat.Phys. 15(6):505 (1976).

[7] Feynmann, R.P. "Statistical Mechanics" Frontiers in Physics Series, D.Pines editor. W.A.Benjamin Inc. Massachussetts (1972).

[8] Borzi, C.H. "Informe de Actualización sobre el Estudio de la Localización de los Separadores de CNE" Internal Report, GAID-CNEA, Argentina (1990).

[9] Leger, M. & Leemans, D.V. "A Study on Hydride Blister Formation in Bruce 3 and 4 and Pickering 5 and 6" Ontario Hydro Research Division 86-153-K Canada (1986).

[10] Otero, D. & Terlisky, S. "Inspection and Evaluation of Pressure Tubes Data at Embalse Nuclear Generation Station" CCTP24 GAID-CNEA, Argentina (1989).
[11] Borzi, C.H. "Localización de los Separadores a lo largo de los Canales Combustibles en Reactores CANDU". Seminar Notes, SUCAN-CNEA, Argentina (1991).

[12] Percus, J.K. Private Communication.

[13] Lebowitz, J.L. & Zomick, D. "Mixtures of Hard Spheres with Non-Additive Diameters. Some Exact Results and Solution of Percus-Yevick Equation" J.Chem.Phys. 54(8):3335 (1971).

[14] Lebowitz, J.L. Phys.Rev. 153:250 (1967).

[15] Kac, M. "Trondheim Theoretical Physics Seminar", Nordita #286 Norway, 1968. There is a good discussion of Kac's method in: Moukarzel C. "Modelos de Vidrios de Spin" Doctoral Thesis IB-CAB-CNEA, Argentina (1991).

[16] Antonaccio, G., Obrusky, L., Martinese, R. and Otero, P. "Informe Final de la Inspección para la Determinación de la Posición de los Anillos Espaciadores del Reactor de CNE" SIMEC-INEND, CNEA report Octubre 1986.

[17] Liu, J. "Span Length versus Required Gap Relationship for Cordoba Reactor" AECL report 31100-200-002, 7-26-88.

[18] Subroutine "IQHSCV" from IMSL,Houston Tx-USA,1980.

TABLE 1: POSITIONS AND DISPERSIONS OF THE GS IN 16 CHANNELS, DISTRIBUTED ON A SECTION OF THE CALANDRIA, MEASURED IN CENTIMETERS FROM ONE OF THE ENDS OF THE CHANNELS. THOSE ROWS WHICH HAVE AN ASTERISK IN THE LAST COLUMN ARE TAKEN FROM REF.[16] AND THE OTHERS FROM REF.[17]. THE CHANNELS ARE NAMED BY THEIR INDEXES (I,J) ON THE INTERPOLATION GRID AS DISCUSSED IN THE TEXT.

CHANNEL	X1	X2	X3	X4	DISP
(1,15)	145.0	242.0	347.0	347.0	2.5
(2,8)	154.0	358.0	358.0	368.0	2.5
(5,20)	138.0	183.0	297.0	450.0	5.0*
(7,8)	104.0	104.0	509.0	509.0	2.5
(7,12)	144.0	222.0	375.0	395.0	2.5
(9,12)	225.0	?	?	265.0	2.5
(9,6)	574.0	574.0	574.0	574.0	2.5
(9,13)	45.0	50.0	195.0	568.0	5.0*
(9,20)	143.0	242.0	347.0	444.0	5.0*
(10,1)	132.0	233.0	336.0	583.0	2.5
(10,7)	145.0	247.0	349.0	584.0	2.5
(10,13)	212.0	247.0	318.0	318.0	2.5
(10,18)	135.0	185.0	365.0	450.0	5.0*
(11,10)	153.0	262.0	335.0	343.0	5.0*
(12,8)	122.0	400.0	550.0	550.0	2.5
(18,9)	223.0	271.0	373.0	576.0	2.5

TABLE 2: PREDICTED VALUES FOR THE POSITIONS AND DISPERSIONS OF THE GS ALONG THE CHANNELS OF TABLE 1 THROUGH THE PROCEDURE DESCRIBED IN THE TEXT. THE NUMERICAL INTERPOLATION WAS PERFORMED BY USING STANDARD ALGORITHMS [18]. THE RELATIVE ERROR (LAST COLUMN) IS DEFINED IN THE TEXT.

CHANNEL	X1	X2	X3	X4	DISP	REL.ERR.
(1,15)	153.5	300.0	355.0	445.0	20.0	0.16
(2,8)	147.5	263.0	265.0	350.0	10.0	0.20
(5,20)	143.0	240.0	350.0	448.0	7.5	0.21
(7,8)	145.0	210.0	247.0	320.0	15.0	0.56
(7,12)	140.0	222.0	372.0	390.0	10.0	0.03
(9,2)	135.0	230.0	335.0	585.0	10.0	0.52
(9,6)	125.0	337.0	400.0	575.0	15.0	0.37
(9,13)	42.0	60.0	198.0	570.0	20.0	0.03
(9,20)	138.0	186.0	365.0	450.0	15.0	0.08
(10,1)	230.0	265.0	330.0	570.0	20.0	0.23
(10,7)	144.0	250.5	350.0	580.0	7.5	0.01
(10,13)	47.0	195.0	350.0	567.0	20.0	0.46
(10,18)	144.0	240.0	345.0	445.0	20.0	0.11
(11,10)	125.0	250.0	320.0	390.0	20.0	0.11
(12,8)	145.0	255.0	347.0	580.0	10.0	0.25
(18,9)	126.0	126.0	402.0	552.0	5.0	0.27