IGNITION OF A MAGNETICALLY CONFINED D-T PLASMA

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ABSTRACT

This study extends Lawson's analysis on the ignition criteria for a magnetically confined plasma by introducing conduction losses from heat transfer theory rather than through the $3nkT/\tau_E$ term (k = Boltzmann constant, T = temperature, τ_E = energy confinement time). It is found that ignition depends on the strength of the magnetic field that confines the plasma, but it is independent of the plasma particle density. As a consequence, an increase of reaction energy through higher particle density leads to higher conduction losses, which can be offset only by higher magnetic fields. It is found that a magnetically confined plasma cannot reach ignition with the magnetic fields considered in present normal size machines.

1. INTRODUCTION

For 40 years, since J. D. Lawson in 1957 published a paper in the Proceedings of the Physical Society by title "Some Criteria for a Power Producing Thermonuclear Reactor,"(1) the plasma physics community has been guided in its quest for fusion by the directions provided by the 'Lawson criteria' for energy breakeven conditions. These are general criteria, applicable to an idealized situation where a plasma of particle density n is brought instantaneously to a temperature T, which is maintained for a time τ_E after which it is allowed to cool to the original temperature. The criteria state that for breakeven, the temperature has to be above 30 million °K and the product $n\tau_E$ must exceed 10^{14} cm^{-3} sec in deuterium-tritium reactions. The reaction products are not retained in the plasma of the Lawson analysis, and conduction losses are neglected during the plasma burning time. These criteria apply to the case of 'scientific breakeven', in which the energy supplied to heat the plasma and maintain the bremsstrahlung losses is returned to the system, together with the reaction energy, with a recovery efficiency η of 1/3. No allowance is made for the fact that the energy used to heat the plasma and supply the bremsstrahlung losses must come from a conventional source, and that the transfer efficiency is normally much less than 100 percent. Were this considered, the Lawson criteria would become even more difficult to satisfy. Mills(2) has generalized the 'scientific breakeven' conditions by considering several energy recovery efficiencies η, from 2.5 to 60 percent. Some discussion regarding terms in the energy balance equation of Lawson has been carried out in Refs. 3 and 4. Several generalizations of the Lawson criteria are also reported in the standard textbook literature on fusion energy[5-11].

In the present work, the complete system is examined in detail, i.e., source of energy and plasma sink, and various energy transfer efficiencies are considered from the first to the second, as well as from the second to the first. The energy from the source is deposited in a pulse, and the plasma experiences a cycle of temperature values. The analysis departs from the conventional one, in that heat conduction losses are introduced from heat transfer theory, rather than through the $3nkT/\tau_E$ term, thus limiting the role played by the energy confinement time in the ignition conditions. In order to ease the inclusion in the energy balance equation of conduction losses, the geometry considered here is the spherical one. More specifically, it is the spherical pinch geometry[12-14]. However, the analysis can be extended to other geometries, including the toroidal, when the appropriate heat conduction loss term is used. The particular case of ignition of a magnetically confined plasma is considered here. Elsewhere, the most general criteria for breakeven or ignition have been provided.[15] A most important result is found that ignition does not depend on particle density. This means that any attempt to have more fusion energy by increasing the fuel (particle) density leads to greater conduction losses. This can be offset by an increase of the magnetic field that confines the plasma. However, it is found that, for a plasma of normal radius, i.e., a centimeter or so, the strength of the magnetic field required for ignition is well above any of the present machines. Also, it is found that the shape of the plasma temperature excursion has a bearing on the ignition conditions.

2. ANALYSIS

Fig. 1 illustrates the system under consideration. Energy E from a source is released in a pulse of power W(t). A fraction E_i of the energy ($E_i = aE$ where a is the efficiency of energy transfer from the source to the sink) is deposited in a spherical chamber containing a plasma core and an ionized gas blanket, while the rest (1 - a)E is dissipated in the components of the energy transfer system. The energy $a\int_o^t W(t)\,dt$

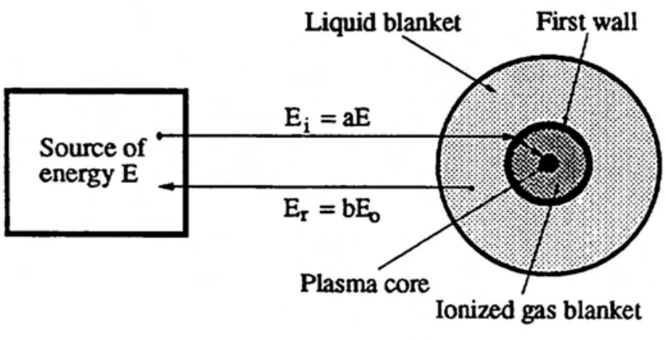

Fig. 1 - Energy flow system.

that reaches the chamber at time t is used to heat the plasma core, and to provide for the radiation and conduction losses. In the absence of other energy inputs, the energy balance equation is:

$$a\int_0^t W(t)\,dt = \int_0^t [H(t) + R(t) + C(t)]\,dt \qquad (1)$$

where H(t) is the rate of heat energy change in the plasma core, and R(t) and C(t) are the rate of energy loss by radiation and heat conduction, respectively. Other losses, such as synchrotron or inverse compton radiation production, are not included here.

A 50 percent mixture of deuterium and tritium is considered here as making up the plasma and surrounding gas. The fusion nuclear reaction taking place in the plasma core is:

$$D + T = He^4\ (3.5\ \text{MeV}) + n\ (14.1\ \text{MeV}) \qquad (2)$$

where 3.5 MeV is the energy carried by the α particle He^4, and 14.1 MeV is the energy carried by the neutron n.

At the end of the life t_o of the plasma core, when this has cooled down and returned to the original temperature, an amount of energy E_o is available from the liquid blanket for return to the source. A fraction E_r of this energy ($E_r = bE_o$, where b is the efficiency of energy transfer from the liquid blanket to the source) is the amount that actually reaches the source. Breakeven or ignition require that $E_r = E$. The aim of this study is to find the conditions for ignition of a magnetically confined plasma as a function of the strength of the confining magnetic field, given the temperature T, particle density n, and radius r of the plasma, and the coefficients a and b of transfer efficiency from the energy source to the sink, and viceversa, respectively, when conduction losses are introduced from heat transfer theory rather than through the $3nkT/\tau_E$ term.

The rate of energy released in the form of α particles and neutrons will be designated as $P_\alpha(t)$ and $P_n(t)$, respectively. Only the α particles may remain in the plasma and further heat it, whereas the neutrons will escape and be absorbed by the liquid blanket. The case considered here is the one in which the α particles are retained (ignition).

<u>α Particles Are Retained</u>

Only the neutrons escape, and the energy available at time t from the liquid blanket for return to the source is:

$$E_o(t) = \int_0^t \left[P_n(t) + R(t) + C(t)\right] dt \qquad (3)$$

Ignition requires that, at the end of the time t_o:

$$\int_0^{t_o} W(t)\,dt = b\int_0^{t_o}\left[P_n(t)+R(t)+C(t)\right]dt \tag{4}$$

Since the α particles remain in the plasma core, this is a new energy source, and the energy balance equation becomes:

$$a\int_0^{t} W(t)\,dt+\int_0^{t} P_\alpha(t)\,dt = \int_0^{t}[H(t)+R(t)+C(t)]\,dt \tag{1A}$$

which can be written as:

$$a\int_0^{t} W(t)\,dt = \int_0^{t}\left[H(t)+R(t)+C(t)-P_\alpha(t)\right]dt \tag{1B}$$

From Eqs. (4) and (1B) one gets:

$$\frac{1}{a}\int_0^{t_o}\left[H(t)+R(t)+C(t)-P_\alpha(t)\right]dt = b\int_0^{t_o}\left[P_n(t)+R(t)+C(t)\right]dt \tag{5}$$

which transforms into:

$$\int_0^{t_o} H(t)\,dt = ab\int_0^{t_o}\left\{P_n(t)+\frac{1}{ab}P_\alpha(t)+\left(1-\frac{1}{ab}\right)[R(t)+C(t)]\right\}dt \tag{6}$$

At the end of t_o, the plasma core returns to the original temperature. No heat energy remains and therefore:

$$\int_0^{t_o} H(t)\,dt = 0 \tag{7}$$

Hence:

$$ab\int_0^{t_o}\left\{P_n(t)+\frac{1}{ab}P_\alpha(t)+\left(1-\frac{1}{ab}\right)[R(t)+C(t)]\right\}dt=0 \tag{8}$$

or

$$\int_0^{t_o}\left\{P_n(t)+\frac{1}{ab}P_\alpha(t)+\left(1-\frac{1}{ab}\right)[R(t)+C(t)]\right\}dt=0 \tag{9}$$

If eq. (9) is satisfied, breakeven is achieved.

The first three terms $P_n(t)$, $P_\alpha(t)$ and $R(t)$ in the integral (9) are functions of plasma temperature, particle density and plasma volume. Specifically, for a 50% D-T mixture, one has[16]:

$$P_n(n, T, V_p) = 8.31 \times 10^{-24} \frac{n^2}{4} T^{-\frac{2}{3}} e^{-\frac{19.94}{T^{1/3}}} V_p \tag{10}$$

$$P_\alpha(n, T, V_p) = 2.06 \times 10^{-24} \frac{n^2}{4} T^{-\frac{2}{3}} e^{-\frac{19.94}{T^{1/3}}} V_p \tag{11}$$

$$R(n, T, V_p) = 2.14 \times 10^{-30} \frac{n^2}{4} T^{\frac{1}{2}} V_p \tag{12}$$

where P_n and P_α are expressed in watts, T in kV, V_p in cm^3, and n in cm^{-3}.

The rate of heat conduction loss C(t) from the plasma core to the liquid blanket can be derived from heat transfer theory[17] as follows. The rate of heat transfer in a hollow sphere with surface temperature T_1 at r_1, and T_2 at r_2, where r_1, r_2 are the radii of the inner and outer surfaces, respectively, is given by:

$$C_1(r_1, r_2, T_1, T_2) = \frac{4\pi K\, r_1\, r_2}{r_2 - r_1}(T_1 - T_2) \tag{13}$$

where K is the thermal conductivity of the material that makes up the hollow sphere.

In the present case, heat transfer takes place from the plasma core of radius r_1 and temperature T_1 to the liquid blanket of radius r_2 and temperature T_2. The medium through which heat propagates before reaching the liquid blanket is the ionized gas blanket made up of deuterium and tritium, and the first wall, usually made up of stainless steel[18]. The latter easily transfers heat, so that only the thermal conductivity of the former needs to be considered. Eq. (13) expresses the rate of heat transfer from the entire volume of the plasma core $V_p = \frac{4}{3}\pi r_1^3$. The rate of heat transfer per unit volume is obtained by dividing (13) by V_p. Since the temperature T_1 of the plasma core is much larger than the temperature T_2 of the liquid blanket, one has $T_1 - T_2 \approx T_1 = T$. Likewise, since $r_2 \gg r_1 = r$, one has $r_2 - r_1 \approx r_2$. Eq. (13) thus leads to:

$$c(r, T) = \frac{4\pi K r_1 r_2 T_1}{(r_2 - r_1)} / \frac{4}{3}\pi r_1^3 \approx \frac{3KT}{r^2} \tag{14}$$

and the heat conduction loss rate appearing in (9) is given explicitly by:

$$C(r, T, V_p) = \frac{3\,K\,T}{r^2} V_p \tag{15}$$

where $C(r, T, V_p)$ is expressed in watts, when T is expressed in kV, r in cm, and K in W/cm kV.

Eq. (15) shows that the rate of heat conduction loss from the plasma core of radius r and temperature T to the liquid blanket is a function of the thermal conductivity K of the ionized gas blanket. In the following, we shall provide the formula for the thermal conductivity of a plasma immersed in a magnetic field.

Plasma Blanket with Magnetic Field. The thermal conductivity is given by[19]:

$$K = 3.54 \times 10^{-25} \frac{A_i^{1/2} Z^2 n_i^2 \ln\Lambda}{T^{1/2} B^2} \frac{\text{cal}}{\text{sec °K cm}} \tag{16}$$

where A_i is the atomic weight of the positive ions of density $n_i = n/2$ (in cm^{-3}), B is the magnetic field (in gauss), and Z = 1 for deuterium and tritium. Taking Ai = 2.5 (averaged for a 50% D-T plasma), and $\ln \Lambda \sim 20$ for a hot plasma, expression (16) transforms into:

$$K = 3.99 \times 10^{-20} \frac{n^2}{T^{1/2} B^2} \frac{\text{W}}{\text{cm kV}} \tag{17}$$

The thermal conductivity in this case decreases with temperature T and magnetic field B, and increases with particle density n. Heat losses are now controlled mainly by the plasma layer with the lowest conductivity. This is the one in contact with the hot plasma core, which can therefore be identified as having the same parameters as the core itself, i.e., the same n and T.

Introducing (17) into (15), one gets:

$$C(r, T, V_p) = 1.61 \times 10^{-19} \frac{n^2 T^{1/2}}{r^2 B^2} V_p \tag{18}$$

If one now inserts the previous expressions for $P_n(t)$, $P_\alpha(t)$, R(t), and C(t) from (10), (11), (12), and (18), respectively, into (9), and divide by V_p, one gets an explicit expression for ignition:

$$\int_0^{t_o} \left\{ \left(8.31 \times 10^{-24} + \frac{2.06 \times 10^{-24}}{ab} \right) \left[\frac{n^2}{4} T(t)^{-\frac{2}{3}} e^{-\frac{19.94}{T(t)^{1/3}}} \right] + \left(1 - \frac{1}{ab} \right) \left| 2.14 \times 10^{-30} \frac{n^2}{4} T(t)^{\frac{1}{2}} + 1.61 \times 10^{-19} \frac{n^2 T(t)^{1/2}}{r^2 B^2} \right| dt = 0 \tag{19}$$

The interesting aspect of this expression is that n^2 appears in all terms of the integrand as common factor, which therefore cancels out. This makes the integrand independent of particle density, and ignition is not a function of n. This is an important result. The integral (19) becomes:

$$\int_0^{t_o} \left\{ \left(8.31 \times 10^{-24} + \frac{2.06 \times 10^{-24}}{ab} \right) \left[\frac{1}{4} T(t)^{-\frac{2}{3}} e^{-\frac{19.94}{T(t)^{1/3}}} \right] + \left(1 - \frac{1}{ab} \right) \left[2.14 \times 10^{-30} \frac{1}{4} T(t)^{\frac{1}{2}} + 1.61 \times 10^{-19} \frac{T(t)^{1/2}}{r^2 B^2} \right] \right. dt = 0 \qquad (19')$$

In (19′) it has been assumed that, during the short time t_o, only the temperature T of the plasma core changes, whereas n and r remain constant. In other words, it is assumed that the plasma is confined by some means during time t_o, and therefore the plasma radius r and particle number density n do not change. The formulation of the conditions for ignition, however, as expressed by (19′), has general validity and remains the same if n and r also are a function of time. It can have even more validity if n is also a function of space within the plasma core volume. In this case, clearly the integration has to be performed over space and over time.

The three functions that appear in the integrals (19′) are positive functions, the latter two being subtracted from the first. The integral can therefore have positive or negative values. If the integrand, which is the sum of these three functions, is negative, the integral will also be negative. Likewise, if it is positive, the integral will be positive. One can examine the trend of the integrand by calculating and plotting it as a function of one of its variables. assuming prescribed values for the others. In this way, one is able to find for which value of the variable the function (integrand) from negative becomes positive, i.e., assumes a value equal to zero. For this value, or for any positive value of the integrand, ignition can be achieved. The geometry to which these results apply is the spherical geometry. For other geometries, such as the toroidal, the proper expressions for the conduction loss must replace the one used here.

In the following figures, plots of the integrand of (19)′ are provided as a function of the magnetic field B. Two parameters are kept constant, namely the coefficient b = 0.3, and the radius of the plasma r = 1 cm. Each figure refers to a specific plasma temperature, ranging from 5 to 20 kV, and contains three plots for a = 0.05, 0.10, and 0.15, respectively.

Fig. 2 shows that, for T = 5 kV, the integrand is always negative, no matter what the value of the magnetic field up to 2 x 10^5 gauss = 20 Tesla. This means that ignition cannot be obtained for the plasma parameters considered in the figure. If one increases the temperature to T = 10 kV, Fig. 3 shows that still ignition cannot be obtained. Only when one pushes the temperature to T = 15 kV (Fig. 4), and to T = 20 kV (Fig. 5), then ignition can be obtained at a value of B around 1.5 x 10^5 gauss (15 Tesla), and B = 1.2 x 10^5 gauss (12 Tesla), respectively. These are very high values of the magnetic field, probably higher than any of those contemplated for future machines.

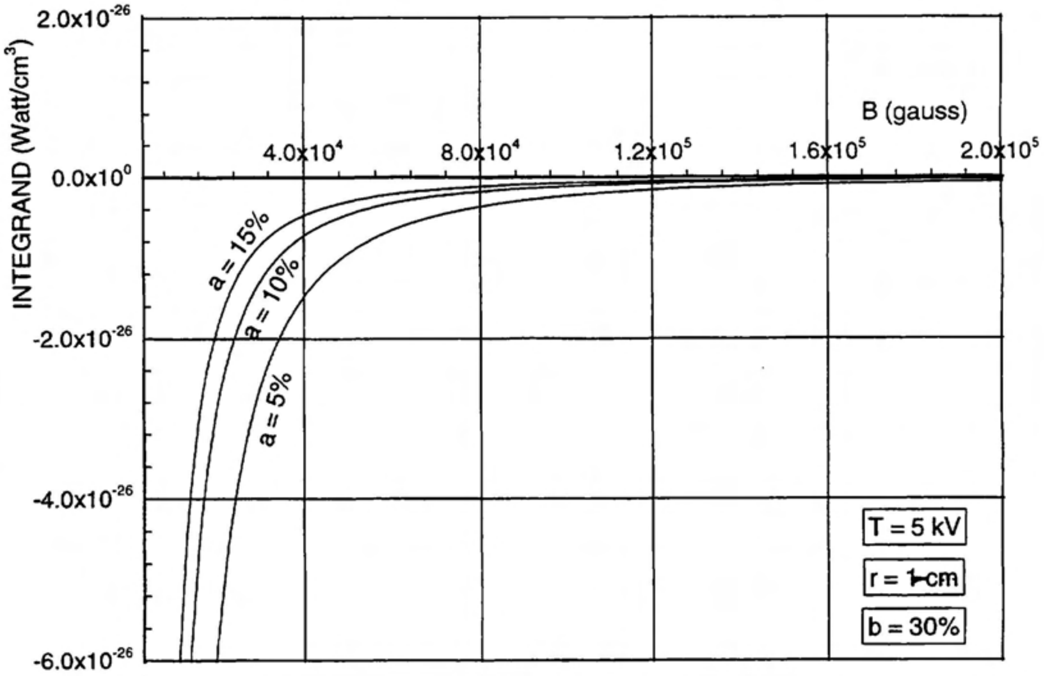

Fig. 2 - Plot of the integrand of Eq. (19′) as a function of confining magnetic field. Ignition cannot be achieved with a confining field of up to 2 x 10^5 gauss (20 Tesla) for a plasma temperature T = 5 kV.

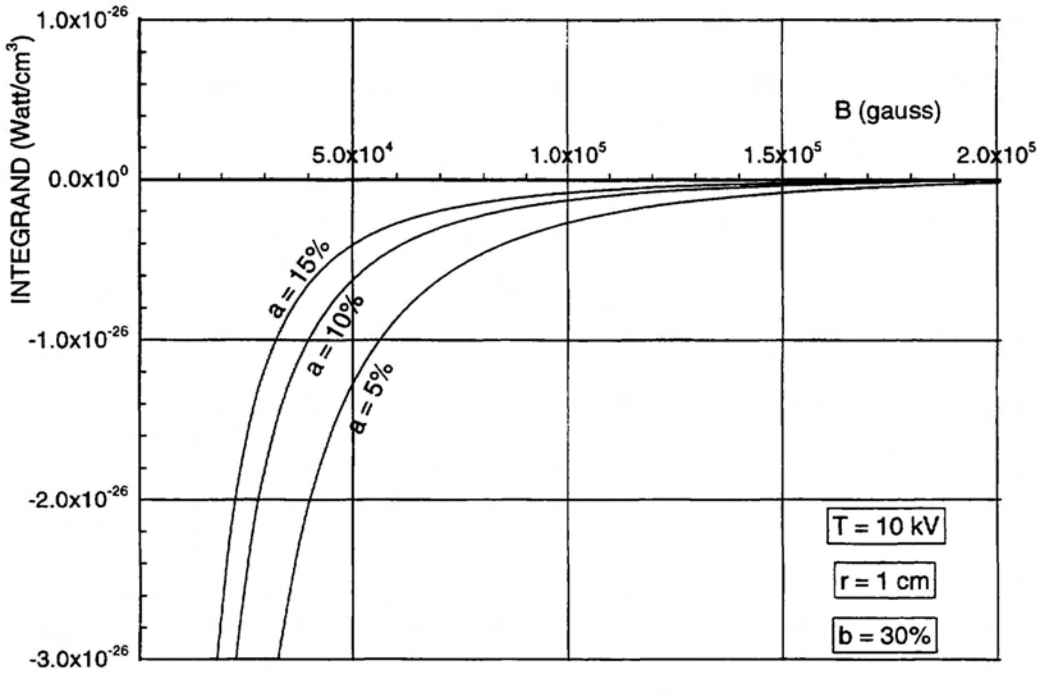

Fig. 3 - Plot of the integrand of Eq. (19′) as a function of confining magnetic field. Ignition cannot be achieved with a confining field of up to 2 x 10^5 gauss (20 Tesla) for a plasma temperature T = 10 kV.

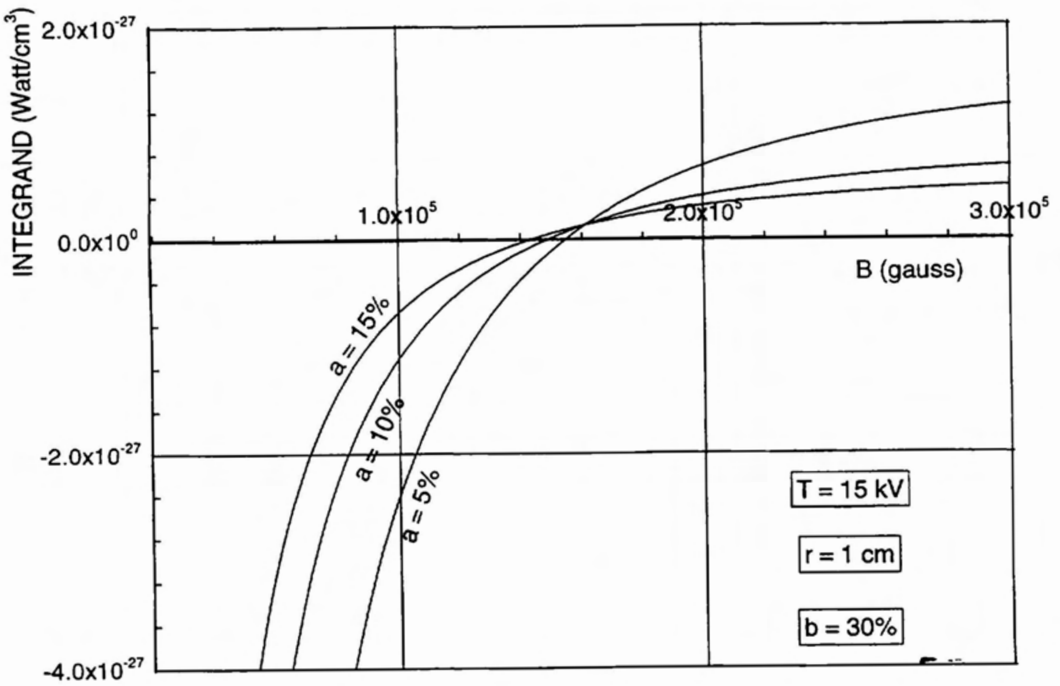

Fig. 4 - Plot of the integrand of Eq. (19′) as a function of confining magnetic field. Ignition can now be achieved with a confining field of 1.5 x 10^5 gauss (15 Tesla) for a plasma temperature T = 15 kV.

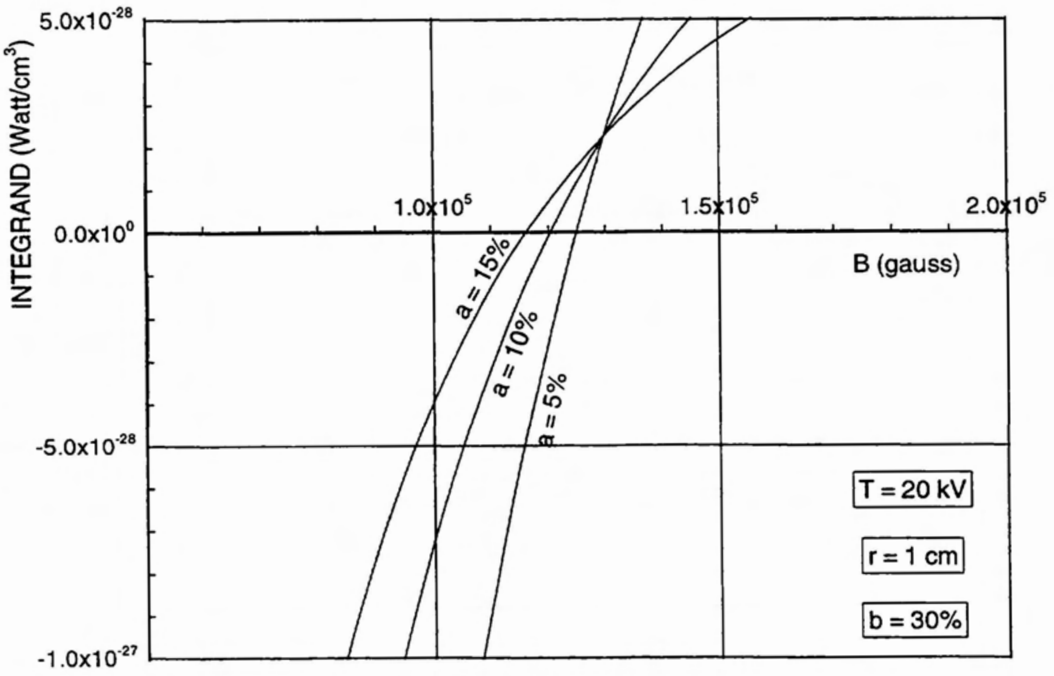

Fig. 5 - Plot of the integrand of Eq. (19′) as a function of confining magnetic field. Ignition can now be achieved with a confining field of 1.2 x 10^5 gauss (12 Tesla) for a plasma temperature T = 20 kV.

The ignition condition derived here as a function of the confining magnetic field is certainly necessary, but by no means sufficient. The reason is that, in a pulsed reactor, the plasma core is subjected to a temperature cycle during the pulse. When the temperature is above threshold for breakeven, there is a gain of energy, and a loss of energy when it is below threshold. The plasma temperature excursion must therefore be shaped in such a way that the energy gained during the time when the temperature is above threshold compensates for the energy lost when it is below this value.

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