DYNAMIC SELF-IGNITION OF A FUSION PLASMA

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ABSTRACT

A new criterion for fusion plasma ignition in d-t tokamaks is established by incorporating non-stationary reaction-thermal dynamics of the plasma into the analysis. Herein the application of a 'soft' Troyon beta limit, together with the actual fusion power deposition and its effect of deteriorating the plasma energy confinement time, are crucial parts of the fusion burn dynamics which determine the ignition conditions. We find that the separatrix established by the dynamic trajectorial evolution of the plasma in the temperature-fuel ion density state plane -- not simply the zero power contour used previously -- is the critical boundary that must be exceeded for the stable ignited operating point to be attained, and maintained, without auxiliary heating.

IGNITION OF A PLASMA

Fusion energy production with tokamak devices demands particular attention to confinement and fuelling regimes that maintain the average fuel ion density $\bar{n}$ and the average plasma temperature $\bar{T}$ at favourable values for optimization of the triple product $\bar{n}\tau_E\bar{T}$, where τ_E denotes the plasma energy confinement time (1). The identification of state and parameter space regions capable of ignited fusion plasma operation is required for significant energy gains to be realized from such systems. This increased energy gain is possible when little or no input power is required to sustain the plasma temperature at a suitable level for fusion reactions to make a useful contribution to the system's energy balance. If ignition is attained, cold fuelling then becomes the sole required injection to the system.

Several definitions of fusion plasma ignition have been considered in the analysis of d-t tokamaks (2,3,4). The requirement that plasma power gains be greater than power losses is consistent, however Ohmic and occasionally auxiliary heating power are included in the power gains. Only Ohmic heating should be so included as the plasma current in a tokamak is always present for plasma stability. While there are numerous references establishing ignited regimes based on steady state power balance calculations, here we consider the non-linear dynamics of the fusion plasma since a steady state power balance alone does not reveal all regions of state space which evolve to the stable ignited operating point.

A globally averaged nonlinear formulation of the dynamic evolutions of density and temperature in an ITER-like d-t tokamak fusion plasma with local temperature $T(\mathbf{r},t)$, and local fuel ion density (deuterons plus tritons) $n(\mathbf{r},t)$ is considered. These state variables are assumed to take on radial profiles which are constant around the toroid (2):

$$T(r,t) = T_o(t)\left(1-\frac{r^2}{a^2}\right)^{\gamma_T}, \qquad n(r,t) = n_o(t)\left(1-\frac{r^2}{a^2}\right)^{\gamma_n}. \tag{1}$$

The peak values of temperature and fuel ion density on the central toroidal axis are $T_o(t)$ and $n_o(t)$ respectively, and r is the radial distance from that axis. The ITER design parameters (5) are used throughout this work; a is the plasma minor radius, $\gamma_T = 1.0$ and $\gamma_n = 0.5$.

REACTION-THERMAL DYNAMICS

The dynamics of plasma fuel ion density evolution are here described on a basis of plasma volume averages by

$$\frac{d\bar{n}}{dt} = \bar{S}(t) - \frac{\bar{n}(t)}{\tau_p} - \tfrac{1}{2}\overline{<\sigma v>_{dt}(r,t)\; n^2(r,t)} \;, \tag{2}$$

where $\bar{S}(t)$ is the volume averaged fuel ion injection rate. The particle confinement time τ_p is taken to be five times the energy confinement time; $<\sigma v>_{dt}$ is the d-t fusion reaction rate parameter. The over-line indicates global averaging over the plasma volume throughout this work. Assuming a constant alpha particle impurity density of $0.03\bar{n}$, the average electron density $\bar{n}_e$ is determined by charge conservation.

The global plasma power balance is formulated as

$$\frac{d(\frac{3}{2}\overline{(n_i+n_e)T})}{dt} = \bar{P}_{aux} + \bar{P}_{ohm} - \frac{\frac{3}{2}\overline{(n_i+n_e)T}}{\tau_E} + \eta_\alpha \bar{P}_\alpha - \bar{P}_{brems} - \bar{P}_{cyc} \;, \tag{3}$$

where n_i is the total ion density including the alpha particle impurities, and $\bar{P}_{aux}$ and $\bar{P}_{ohm}$ are the auxiliary and ohmic power densities input to the plasma, respectively. The fraction of the d-t fusion alpha power density, $\bar{P}_\alpha$, retained in the plasma is η_α, while $\bar{P}_{brems}$ and $\bar{P}_{cyc}$ are the bremsstrahlung and cyclotron radiation power density losses from the plasma. Recent analyses of alpha particle thermalization and energy deposition to the plasma (6) revealed a difference of less than 1 keV between the ion and electron temperatures on the time scale of the dynamic (7) unless there is substantial preferential auxiliary heating of one species. Thus, a single temperature formulation was deemed sufficient. Formulations for the terms on the right-hand-side (RHS) of Eq.(3) are as follows, with the power expressions in units of $keV \cdot s^{-1} \cdot m^{-3}$.

The globally averaged ohmic heating power density due to the induced toroidal plasma current is here taken to be (8)

$$\bar{P}_{ohm} = \frac{1.7\times 10^{19} Z_{eff} I^2}{a^4 \kappa^2 \bar{T}^{3/2}} \;, \tag{4}$$

where I is the plasma current (MA), and κ the plasma elipticity (5). The effective charge of the plasma, Z_{eff}, is defined by (1)

$$Z_{eff} = \frac{\sum_j \bar{n}_j Z_j^2}{\bar{n}_e} \;, \tag{5}$$

where $\bar{n}_j$ is the average particle density and Z_j the charge number of the j-th type ion.

The local bremsstrahlung radiation power density losses are (1)

$$P_{brems} = 3.346\times 10^{-21} n_e^2 \sqrt{T}\,[Z_{eff}(1+0.00155T+7.15\times 10^{-6}T^2) + \frac{0.071 C_1}{\sqrt{T}} + 0.00414T\,] \;, \tag{6}$$

where C_1 is a similar quantity to Z_{eff} , the former given by (1)

$$C_1 = \frac{\sum_j \bar{n}_j Z_j^3}{\bar{n}_e} \ . \tag{7}$$

Local cyclotron radiation losses are taken as (1)

$$P_{cyc} = 0.3878 n_e T B^2 \phi \ , \tag{8}$$

where B is the toroidal magnetic field (Tesla),

$$\phi = \frac{0.005198}{\sqrt{\Lambda}} T^{3/2} \sqrt{1 + \frac{22.61 a}{R\sqrt{T}}} \sqrt{1 - R_f} \ , \tag{9}$$

and

$$\sqrt{\Lambda} = 7.78 \times 10^{-9} \sqrt{\frac{n_e\ a}{B}} \ . \tag{10}$$

The plasma major radius is R, and R_f is the global cyclotron radiation re-absorption fraction -- taken to be 0.9 herein.

The globally averaged d-t fusion alpha power density is evidently

$$\bar{P}_\alpha = \frac{3517}{4} \overline{<\sigma v>_{dt} n^2} \ . \tag{11}$$

For the global energy confinement time, the ITER scaling law (9)

$$\tau_E = \frac{0.048 f_e \sqrt{\kappa M} a^{0.3} R^{1.2} I^{0.85} B^{0.2} \bar{n}_e^{\ 0.1}}{\sqrt{\bar{P}}} \ , \tag{12}$$

was used, where f_e is the h-mode energy confinement time enhancement factor, M the average isotopic mass of the plasma, $\bar{n}_e$ is in 10^{20} m^{-3}, and $\bar{P}$ the total power (MW) contributing to the degradation of confinement, explicitly:

$$\bar{P} = \bar{P}_{aux} + \bar{P}_{ohm} + f_\alpha \eta_\alpha \bar{P}_\alpha \ . \tag{13}$$

The fraction of d-t fusion alpha power which degrades confinement is taken here to be $f_\alpha = 0.5$, and further $\eta_\alpha = 0.95$ (6). Combining Eqs.(12) and (13), evaluating the ITER parameters and introducing a soft beta limit (2) yields

$$\tau_E = \frac{1.056 \times 10^9\ \bar{n}_e^{\ 0.1}}{SBL\sqrt{\bar{P}_{aux} + \bar{P}_{ohm} + f_\alpha \eta_\alpha \bar{P}_\alpha}} \ , \tag{14}$$

with $\bar{n}_e$ now in m^{-3}.

A soft beta limit of the form

$$SBL = e^{\left(\frac{\beta}{0.85\beta_c}\right)^{10}} , \tag{15}$$

was used to model confinement degradation near the Troyon limit, or critical beta

$$\beta_c(\%) = \frac{C_{Troy} I}{aB} , \tag{16}$$

in an inhibitive manner which differs from that found in the literature (2,10). C_{Troy} is the Troyon factor (2). Specifically, confinement degradation was made to begin at ~80% of β_{crit} and increase exponentially such that transport losses were increased by a factor > 10^2 at the Troyon limit. It is expected that such enhanced transport losses will occur due to the increasing kinetic pressure within the tokamak as the beta limit is approached and not only if β_{crit} is surpassed.

CALCULATIONAL ASSESSMENT

Taking the burn characteristics of the d-t fusion plasma as determined by the temporal evolutions of $\bar{n}(t)$ and $\bar{T}(t)$, we examine the associated dynamics in a state plane spanned by the state variables $\bar{T}$ and $\bar{n}$. Two stationary points of the system dynamic (the solution of the RHS of both Eq.(2) and Eq.(3) = 0 with $\bar{P}_{aux} = 0$) -- one an attractor and the other a saddle point are shown in Figure 1. Also depicted are the regions of the $\bar{T}$-$\bar{n}$ plane for which there is a positive steady state power balance (RHS of Eq.(3) > 0 with $\bar{P}_{aux} = 0$) -- bounded by the zero power contours, all for an ion injection rate of $\bar{S} = 0.05 \times 10^{20}$ $m^{-3} \cdot s^{-1}$. The latter region is the area of the state plane where the net power deposited in the plasma exceeds the total power losses, and thus where ignited plasma operation would appear possible. The Ohmic ignition point ($\bar{n} > 10^{21}$ m^{-3}) is not shown as it is of no interest here due to the insignificant fusion power.

However, of greater interest is the dynamic evolution of the system determined by solving Eqs.(2) and (3) for $\bar{n}(t)$ and $\bar{T}(t)$, yielding the trajectories and separatrices of Figure 1. Evidently, a high temperature, low density region from which the stable ignited point is attained dynamically exists outside the zero power contours. This region of 'dynamic ignition' is not foreseeable solely through power balance ignition studies, and thus alters the regimes of the $\bar{T}$-$\bar{n}$ state plane which are ignited -- those plasma states which do not require auxiliary heating or control to evolve to, and remain at, the stable high temperature attractor.

Conversely, the region below the separatrix but within the positive steady state power balance region does not ignite as the system's dynamics result in a trajectorial evolution to the low temperature, high density, Ohmic operating point. This further alters the conventional ignited regions of state space, and so the ignition criterion for an ITER-like plasma is not merely achieving a plasma state within the positive steady state power balance region, but rather the separatrix of Figure 1 must be exceeded for the system to evolve dynamically to its high temperature attractor. The location and nature of this attractor has elsewhere been shown (10) to be dependent upon the fuel ion injection rate.

The region of dynamic ignition suggests a possible new approach to heating a fusion plasma to ignited conditions. If attaining a plasma state within this region of state space is less energy demanding than attaining one within the steady state-defined ignition region, a less energy intensive method to achieve plasma ignition is evident.

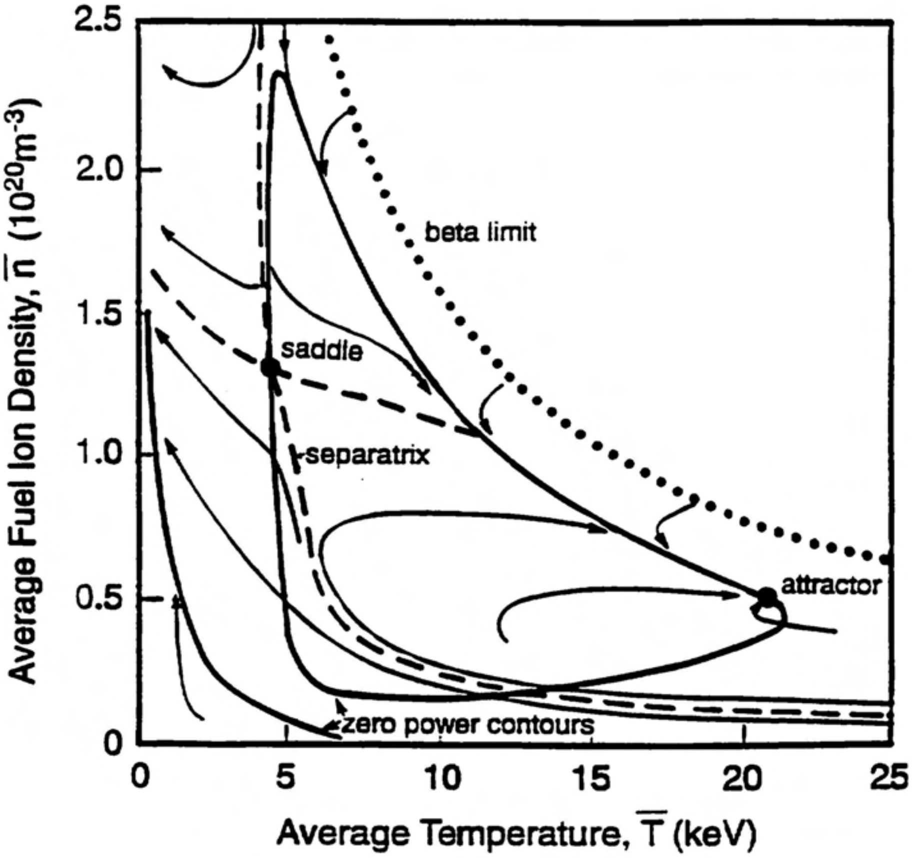

Figure 1: Steady state power balance ignition regimes of the $\bar{T}$-$\bar{n}$ state plane bounded by the zero power contours ($\bar{P}_{aux}$ = 0) and the dynamic ignition regimes defined by the separatrix deduced from state plane trajectories ($\bar{S}$ = 0.05 × 10^{20} $m^{-3} \cdot s^{-1}$).

CONCLUDING COMMENTS

We thus conclude that the criterion for ignited operation of a fusion plasma is the crossing of the separatrix in the $\bar{T}$-$\bar{n}$ state plane -- a consequence of the non-linear dynamics of the system -- as the power balance alone does not completely reveal which regions of state space evolve dynamically to the stable ignited attractor. A region of 'dynamic self-ignition' outside the positive power balance region was identified and the dynamic ignition criterion of exceeding the separatrix must be recognized as an important consideration for both initial plasma heating and ignited operation of a d-t tokamak.

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