

MODELING EXCAVATION RESPONSE
IN GEOLOGICAL REPOSITORIES FOR RADIOACTIVE WASTE
USING DISCONTINUOUS DEFORMATION ANALYSIS METHOD

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INTRODUCTION

A final disposal program for the spent nuclear fuel has been conducted in Taiwan for near 10 years. At this stage, the program is processed with the regional investigation of the second year task of Phase III, which was scheduled for 10 years. ITRI has been authorized by Taiwan Power Company to conduct this program for the first 4 years. The main goals are: (1) to develop the techniques and test equipments for the regional investigation, (2) to conduct researches on thermal properties of host rocks, engineering characteristics of hydraulic barriers, and retardation/dispersion of radionuclide for the program of performance/safety assessment, and (3) to complete the preparatory work for the execution of site investigation in the next 6 years. In order to safely dispose of the spent nuclear fuel, the concept of Deep Geological Disposal adopted by the other countries has been selected in this program.

In this program, the project of rock mechanics is to be initiated this year. The work will be on reviewing papers of the in-situ stress measuring techniques and proposing the future plan of the project. The techniques of numerical modeling for rock engineering are also under our plans to develop.

Rock masses, in general, are not continuous. Therefore, continuum models, such as Finite Element Method (FEM) and Boundary Element Method (BEM), usually do not work well when predicting the response of rock masses to loading and unloading. On the other hand, Discrete Element Methods (DEM) are tailored for problems with many discontinuities and large displacements. The Discontinuous Deformation Analysis (DDA) method is a recently developed technique that falls into the family of DEM. Large displacements and deformations are considered under both static and dynamic loadings.

Dr. Shi first presented the DDA method in 1984 [1] and published his PhD thesis [2] in 1988. Two models were included in the method. First, the backward model computes the strains and displacements of a blocky system that best explains a set of displacement and strain observations made at a sufficient number of points. Second, the forward model computes the stresses, strains, sliding, block contact forces and block movements of a blocky system, based upon the knowledge of geometry, loading, and material constants (Young's moduli and Poisson's ratio) of each block,

as well as friction angles of contact joints.

The distinct feature of the forward analysis, allowing large movements of rock blocks, enables the study of a variety of applications in rock mechanics including stability analysis and support design of underground excavations, dam abutments, and rock-fall [2]. This forward analysis has been used in several fields of rock mechanics, such as fracture propagation in intact rock using the artificial joint concept by Ke and Goodman [3], fracture analysis in intact rock and jointed rock using sub-block method by Lin [4], dynamic rock failure using simplex integrations by Shi [5], and the behavior of tunnel openings in jointed rock masses by Yeung and Klein [6].

THE DDA METHOD

In the DDA method, the displacements (u,v) at any point (x,y) in a block, i , are represented, in two dimensions, by three displacements and three strains, usually denoted in vector form by

$$\mathbf{D}_i = (d_{1i}, d_{2i}, d_{3i}, d_{4i}, d_{5i}, d_{6i})^T = (u_o, v_o, r_o, \epsilon_x, \epsilon_y, \gamma_{xy})^T \quad (1)$$

where (u_o, v_o) is the rigid body translation at a specific point (x_o, y_o) within the block, r_o is the rotation angle of the block with a rotation center at (x_o, y_o) and ϵ_x , ϵ_y and γ_{xy} are the normal and shear strains in the block.

Assuming first order approximation, the displacements (u,v) at a point (x,y) inside block i can be expressed as follows

$$\begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{T}_i \mathbf{D}_i \quad (2)$$

where

$$\mathbf{T}_i = \begin{bmatrix} 1 & 0 & -(y-y_o) & (x-x_o) & 0 & (y-y_o)/2 \\ 0 & 1 & (x-x_o) & 0 & (y-y_o) & (x-x_o)/2 \end{bmatrix} \quad (3)$$

This equation enables the calculation of the displacements at any point (x,y) within the block (in particular, at the corners), when the displacements are given at the center of rotation and when the strains (constant within the block) are known. In the two dimensional formulation of the DDA method, the center of rotation with coordinates (x_o, y_o) coincides with the centroid with coordinates (x_c, y_c) .

In the DDA method, individual blocks form a system of blocks through contacts among blocks and displacement constraints on single blocks. Assuming that n blocks are defined in the block system, Shi [2] showed that the simultaneous equilibrium equations can be written in matrix form as follows

$$\begin{bmatrix}
 K_{11} & K_{12} & K_{13} & \dots & K_{1n} \\
 K_{21} & K_{22} & K_{23} & \dots & K_{2n} \\
 K_{31} & K_{32} & K_{33} & \dots & K_{3n} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 K_{n1} & K_{n2} & K_{n3} & \dots & K_{nn}
 \end{bmatrix}
 \begin{bmatrix}
 D_1 \\
 D_2 \\
 D_3 \\
 \vdots \\
 D_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 F_1 \\
 F_2 \\
 F_3 \\
 \vdots \\
 F_n
 \end{bmatrix}
 \quad (4)$$

where the total number of displacement unknowns D_{ij} is the sum of the degrees of freedom of all the blocks. In form, the system of Equation (4) is similar to that in Finite Element Method FEM. Mathematically, the system of equations (4) is solved for the displacement variables like FEM. However, the solution is constrained by a system of inequalities associated with block kinematics (e.g. no penetration and no tension between blocks) and Coulomb friction for sliding along block interfaces. The final solution to Equation (4) is obtained as follows. First, the solution is checked to see how well the constraints are satisfied. If tension or penetration is found along any contact, the constraints are adjusted by selecting new locks and constraining positions and a modified version of K_{ij} and F_{ij} are formed from which a new solution is obtained. This process is repeated until no tension and no penetration is found along all of the block contacts. Hence, the final displacement variables for a given time step are actually obtained by an iterative process.

The simultaneous equations (4) were derived by Shi [2] by minimizing the total potential energy, Π , of the block system. The i -th row of Equation (4) consists of six linear equations

$$\frac{\partial \Pi}{\partial d_{ri}} = 0, \quad r=1-6 \quad (5)$$

where the d_{ri} are the deformation variables of block i . The total potential energy is the summation over all the potential energy sources, that is, individual stresses and forces. The potential energy of each force or stress and their derivatives are computed separately. The derivatives

$$\frac{\partial^2 \Pi}{\partial d_{ri} \partial d_{sj}}, \quad r, s=1-6 \quad (6)$$

are the coefficients of unknowns d_{sj} of the equilibrium equations (4) for variable d_{ri} . All terms of Equation (6) form a 6 x 6 sub-matrix, which is sub-matrix K_{ij} in Equation (4). Equation (6) implies that matrix K in (4) is symmetric. The derivatives

$$-\frac{\partial \Pi(0)}{\partial \vec{d}_{ri}}, \quad r=1-6 \quad (7)$$

are the free terms of Equation (5) which are shifted to the right hand side. All these terms form a 6 x 1 sub-matrix, which is added to the sub-matrix F_i .

Dr Shi's thesis [2] covers the details for forming sub-matrices K_{ij} and F_i , for elastic stresses, initial stresses, point loads, line loads, volume forces, bolting forces, inertia forces and viscous forces.

THE CRITERION OF BLOCK FRACTURING

The criterion selected in this paper for block fracturing is a Mohr-Coulomb criterion with three parameters : s_0 is the inherent shear strength of the block material, ϕ is its friction angle and T_0 represents its tensile strength. It is assumed that tensile normal stresses are positive, and the major and minor principal stresses are denoted as σ_1 and σ_3 (with $\sigma_1 \geq \sigma_3$), respectively. A critical value of the minor principal stress is defined as

$$\sigma_{3c} = -C_0 + T_0 \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \quad (8)$$

where $C_0 = 2s_0 \tan(\pi/4 + \phi/2)$ is the unconfined compressive strength of the block material. According to the Mohr-Coulomb fracturing criterion, shear failure occurs when

$$\sigma_3 < \sigma_{3c} \quad \wedge \quad \sigma_3 \leq -C_0 + \sigma_1 \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \quad (9)$$

(where ' $\wedge$ '='and') and tensile failure occurs when

$$\sigma_3 \geq \sigma_{3c} \quad \wedge \quad \sigma_1 \geq T_0. \quad (10)$$

The immediate advantage of this criterion is that different types of fracture (in tension or shear) are well defined by the transitional normal stress σ_{3c} . Tensile failure is more likely to occur in strong brittle rocks under tension. On the other hand, shear failure is more likely to occur in weak rocks.

The three-parameter Mohr-Coulomb criterion was added to the DDA program and is graphically shown in the theoretical and physical plots in Figures 1(a) and 1(b). For each block of the system, the major and minor in-plane principal stresses σ_1 and σ_3 are determined at the block's centroid. If the condition of shear failure is satisfied within a breakable block, two failure planes are introduced. They pass through the block's centroid and inclined at $\mp (\pi/4 - \phi/2)$ with respect to σ_3 , as shown in Figure 1. Then, the block is divided into four blocks and the analysis is

resumed with a new block configuration. If, on the other hand, the condition of tensile failure is satisfied within a breakable block, a failure plane is introduced. It passes through the block's centroid and oriented at right angles to σ_1 as shown in Figure 1. In this case, the block is divided into two blocks and the analysis is resumed.

In this formulation, no energy dissipation is assumed to occur during shear or tensile failure. Upon breaking, the new blocks are assumed to have the same velocities as the original block. Also, the new fractures have Coulomb friction and cohesion.

MODELING UNDERGROUND EXCAVATION IN ROCKS

Underground excavations are used in a variety of engineering projects. Underground excavations can be tunnels (railway, highway, etc...) or caverns (underground power plants, storage caverns, nuclear waste repositories, etc...). Predicting the response of a rock mass to excavation can be done using closed-form (analytical) solutions or numerical methods. Closed-form solutions are only suited for ideal linear elastic and homogeneous rock conditions and for excavations of regular geometrical shape. On the other hand, for rock masses with more complex constitutive behavior or for excavations of complex geometry, numerical methods are necessary. The numerical methods that have been used for the modeling of underground excavations in rock include : the BEM method, the FEM method [7,8,9], the coupled FEM and BEM (FEBEM) method [10,11,12], and the Discrete Element Method [13]. Among those, the BEM, FEM, and FEBEM methods can be used to determine the displacements and stresses in rock masses that are either continua or discontinua with a limited number of discontinuities. Displacements along those discontinuities must be small and block movement is limited. On the other hand, in the DEM approach, the deformations within blocks and the kinematic motions across discontinuities are separately treated and therefore the mechanical behavior of jointed rock masses can be directly modeled. Also, large displacements and deformations in a rock mass are allowed. However, this method requires special treatment to account for the interaction among individual blocks.

In the next three examples, the isotropic elastic stress and strain relationship in plane strain has been used to model the block constitutive behavior and the static analysis has been used to model the block movements and deformations. The penalty method has been used to enforce the block contacts.

Example 1. Consider a 4m x 5m tunnel constructed in 14m x 15m jointed rock masses, as shown in Figure 2(a). The joint spacing is 1.5m except the horizontal joint spacing in the middle is 2m. A uniform vertical stress of 17.3 MPa is applied on top of the model to simulate the overburden of 667m. The rock mass is not free to deform laterally. Fixed boundary block is used to simulate no lateral displacement. The intact rock has a unit weight $\gamma=2.6 \times 10^{-2}$ MN/m³, a Young's modulus $E=1,000$ MPa, and a Poisson's ratio $\nu=0.23$. The joint friction angle is 15 degrees. The analysis result after 1,500 time steps of 0.001 second time increment is shown Figure 2(a) and (b). Figure 2(a) shows the deformed tunnel (with solid lines) after its excavation. The bottom of the deformed tunnel exhibits an apparent heaving behavior which has been often found in the underground excavation. Figure 2(b) shows the size of the disturbed zone is about 2m from the side walls, about 4m above the roof, and more than 4m below the bottom.

Example 2. Consider again the same block geometry, material properties, and loading condition, as described in Example 1. Two rock bolts are placed on the roof block. Figure 3(a) shows the deformed tunnel after its excavation. Figure 3(b) shows that the disturbed zone is greatly changed around the tunnel comparing to the one in Figure 2(b). In other words, the stress concentration area and magnitude around the tunnel is much smaller than the one in Example 1. It demonstrates that the rock bolts are able to stabilize the tunnel structure. As the rock bolts were not placed on the bottom of the tunnel, the heaving behavior was not eliminated there.

Example 3. Consider again the same block geometry, material properties, and loading condition, as described in Example 1, except the existence of a fault at a dip of 68.2 degrees, as shown in Figure 4(a). Figure 4(b) shows the disturbed zone is abruptly changed around the tunnel and along the fault. Especially, a stress concentration area exists on top of the tunnel and along the fault. Again, as the rock bolts were not placed on the bottom of the tunnel, the heaving behavior was still found there.

CONCLUSIONS

It is a new attempt to apply the DDA method to model the underground excavation problems which will be encountered in the final disposal program for the spent nuclear fuel in Taiwan. The method, basically, allows large displacements along rock joints and enables to simulate block kinematic motions. The examples, as presented in the previous section, demonstrate the ability of the method to capture the disturbed zone around tunnels with the existence of rock bolts or located in various geological conditions of rock mass.

The newly developed program for the DDA method by Lin [4] also enables to simulate the fracture analysis of rocks. One example, as shown in Figure 5, shows a tunneling problem with a roof failure using the DDA dynamic analysis. Prior to block fracturing, the rock blocks were uniformly in square shape and supported by a beam as the top of the 3.7m x 4.9m tunnel. However, the rock mass collapses when the support beam starts breaking. More examples allowing block fractures can be found in reference [4].

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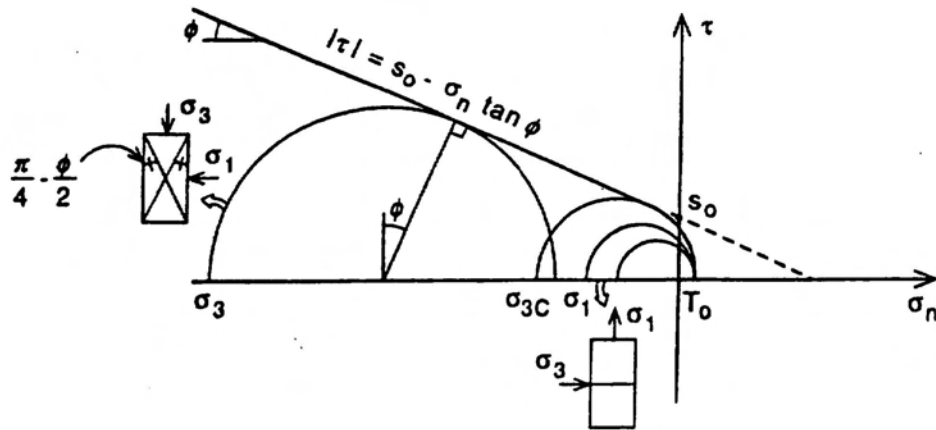


Figure 1(a)

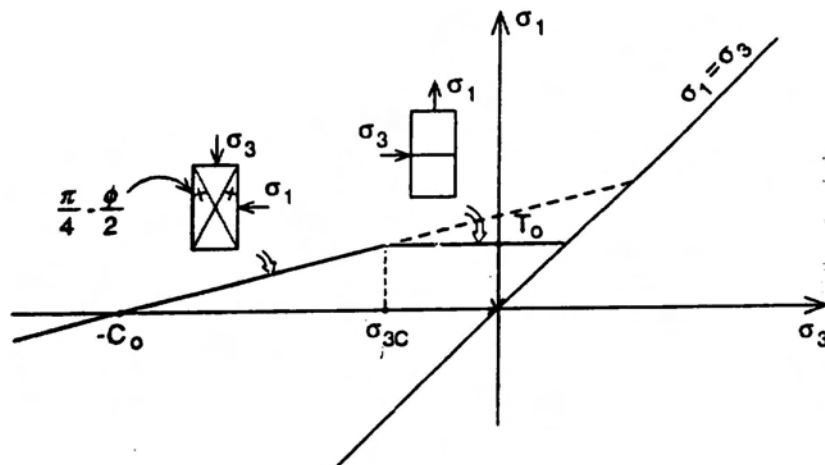


Figure 1(b)

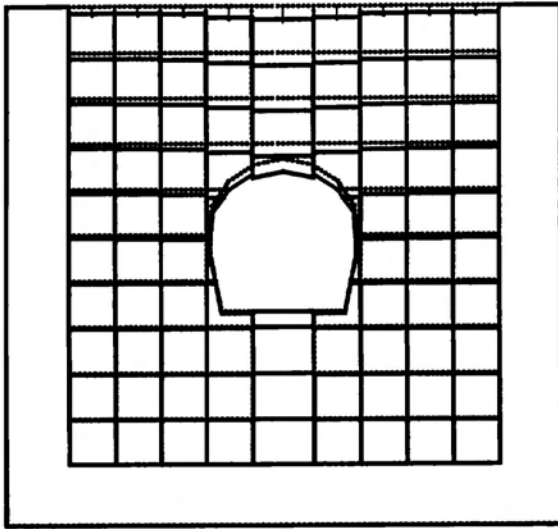


Figure 2(a)

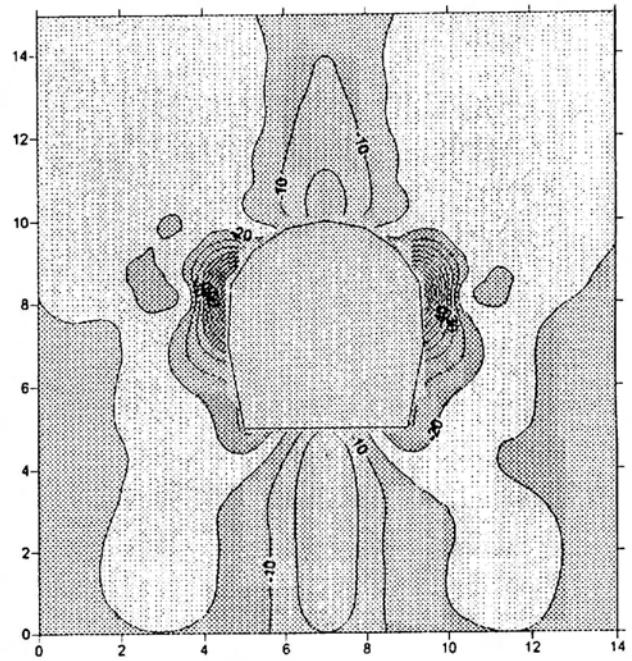


Figure 2(b)

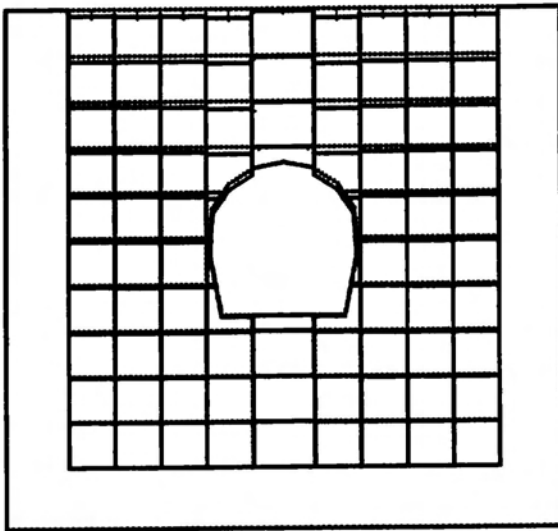


Figure 3(a)

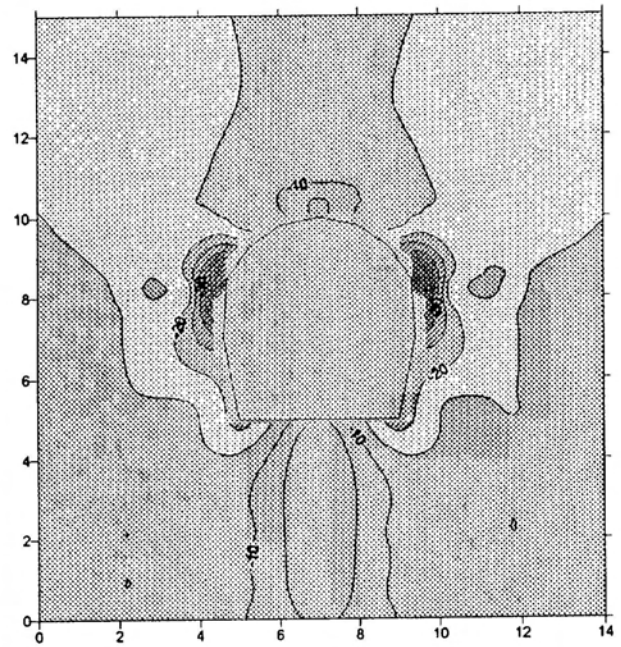


Figure 3(b)

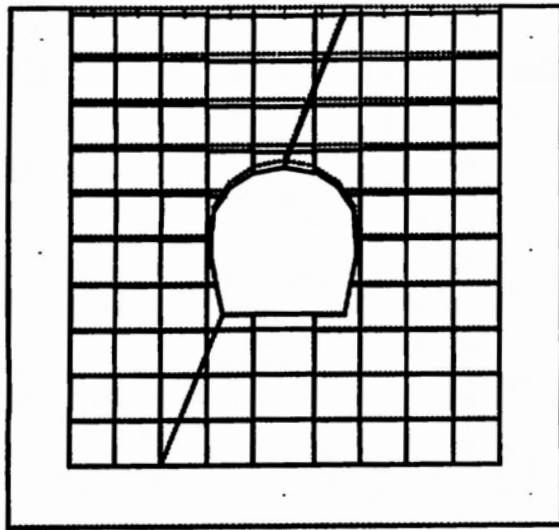


Figure 4(a)

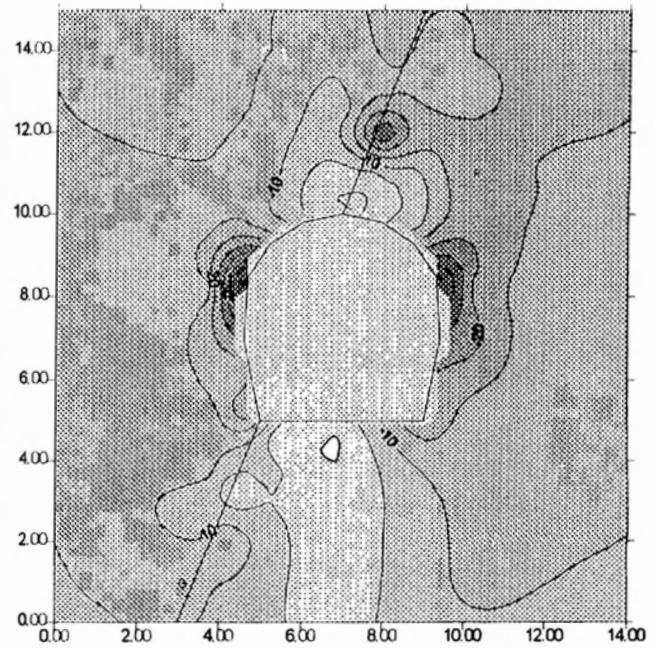


Figure 4(b)

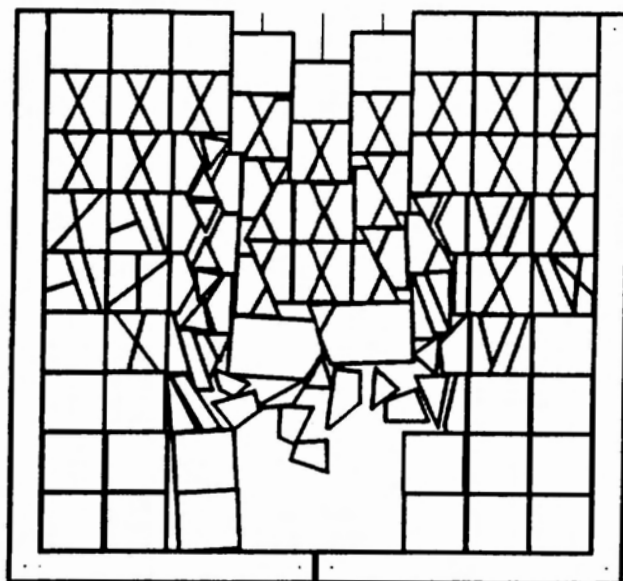


Figure 5