A Probabilistic Approach to Channel-Power-Limit and Bundle-Power-Limit Compliance Analysis

by

F. A. R. Laratta and H. C. Chow

Atomic Energy of Canada Limited
2251 Speakman Drive
Mississauga, Ontario L5K 1B2

and

E. G. Young, C. W. Newman and R. E. Stone

New Brunswick Power
Point Lepreau Nuclear Generating Station
P.O. Box 10
Lepreau, New Brunswick E0G 2H0

ABSTRACT

The formulation of a probabilistic approach to channel-power-limit and bundle-power limit compliance analysis is described. The performance indicator is the probability that no channel (bundle) exceeds its limit. In contrast to monitoring only the maximum power channel and the probability that the maximum channel power does not exceed the limit, the proposed approach accounts simultaneously for all channels. Xenon transients in the fresh bundles in the refuelled channels are treated rigorously. The time variation of xenon build-up is accounted for according to the elapsed time since refuelling for each of the refuelled channels. Instantaneous probability is then computed. Average probabilities over the time interval between surveillance calculations can also be computed, as well as time-integrated probability below certain targets. The proposed method has been demonstrated in sample applications using Point Lepreau Generating Station operation data. Sensitivity to key input uncertainty is also illustrated.

1. INTRODUCTION

CANDU plants operate within certain limits on maximum channel power and bundle power. Compliance with these limits is demonstrated in CANDU 6 plants, through simulations of reactor operation, performed at frequent intervals, using reactor physics codes and models. It is recognized that, in these calculations, there are inherent uncertainties that must be taken into account when the calculated powers are compared with the limits. Also, there are power variations between surveillance times, caused by operational manoeuvres such as channel refuelling. These variations must be considered in compliance demonstration.

The calculation method currently used at the Point Lepreau Generating Station (PLGS) is flux and power mapping using the RFSP code [1]. The mapping method is based on flux synthesis, using a set of flux-shape mode functions and forcing a best fit to the readings of 102 in-core vanadium detectors. There are inherent uncertainties in the mapped powers because of various approximations in the methodology and because of detector measurement errors. There are also uncertainties introduced in normalization to the total reactor power. It is important that the magnitude of these uncertainties be carefully determined and justified on the basis of comparisons with measurement data. An extensive error assessment has been performed and has culminated in the production of an elaborate error model which distinguishes uncertainties and biases in the mapped power for nominal and off-nominal conditions.

Administrative action points are set currently at one standard deviation and two standard deviations (based on the total RFSP uncertainty in the mapped power) below the limits and are used to monitor the reactor power history. Excursions beyond these administrative action points are tracked. The PLGS internal practices specify the number of times these action points are allowed to be reached and the response time within which corrective action is to be taken.

The current compliance analysis method is deterministic in the sense that the administrative action points are set at certain margins below the limits, and excursions beyond these administrative action points are tallied. There are no quantitative indicators of the probability of exceeding the limits for a given instantaneous snapshot power distribution. In this paper, a possible probabilistic approach to establish such indicators is described. This approach draws on the Reactor Overpower Protection Trip (ROPT) probability calculation methodology. It computes the channel and bundle power probability that no channel (or bundle) exceeds its limit. The framework of this approach has been incorporated in a new calculational module (*COMPROB) in RFSP for trial applications.

2. BASIC METHODOLOGY

The current probabilistic compliance framework has been formulated to accommodate three error sources and uncertainties:

- the uncertainty and bias due to bulk power normalization, common to all channels and bundles, called respectively the "common random" uncertainty and "common bias" error. The "common random" uncertainty is sometimes called the "time random" uncertainty;
- the RFSP power mapping independent channel random uncertainties (and similarly for bundles), these are sometimes termed "spatial random" uncertainties;
- the systematic biases due to transient refuelling (xenon) effects and uncertainty in such biases for channels and bundles affected by refuellings that have recently taken place.

We shall describe the probabilistic method for channel power. The method is similar for bundle power. The objective is to determine, for a given instantaneous channel-power distribution as computed in the surveillance mapping calculation, the probability that no channel exceeds its limit. Note that at PLGS, the limit is channel specific: the central high-power channels have a limit of 7300 kW, and the outer channels have lower limits based largely on the premise that the same margin to dryout is maintained.

We define

CP_i	as the mapped power for channel i;
$CPLIM_i$	as the power limit for channel i;
μ_i	as the total bias error for channel i , including the common bias error; and
σ_i	as the one-sigma channel random uncertainty of the computed CP_i .

The bias-corrected channel power is

$$CPP_i = CP_i * (1 + \mu_i) \tag{1}$$

The margin to the limit for channel i is then $(CPLIM_i / CPP_i - 1.0) * 100\%$.

For the time being, we ignore the common random uncertainty. Assuming that the channel random errors follow a Gaussian distribution, the probability that CPP_i is less than $CPLIM_i$ is given by

$$Pr_i = 1 - \frac{1}{\sqrt{2\pi}\sigma_i * CPP_i} \int_{-\infty}^{CPP_i} e^{-\frac{1}{2}(dv)_i^2} dZ \tag{2}$$

where Z is a running variable (channel power) that appears in the deviate $(dv)_i$, which is defined as

$$(dv)_i = (Z - CPLIM_i) / (CPP_i * \sigma_i) \tag{3}$$

For computational purposes, we extend the evaluation of Pr_i to cover a range of reactor powers (xr) covering $\pm$ 10% of full power; that is, we compute $Pr_i (xr*CPP_i)$ with xr covering the range 0.9 to 1.1 .

Using a transformation $z = Z / CPP_i$, the probability that $xr*CPP_i$ is less than $CPLIM_i$ is then given by

$$Pr_i (xr) = 1 - \frac{1}{\sqrt{2\pi}\sigma_i} \int_{-\infty}^{xr} e^{-\frac{1}{2}(dv)_i^2} dz \tag{4}$$

The deviate $(dv)_i$ is now

$$(dv)_i = \frac{z * CPP_i - CPLIM_i}{CPP_i * \sigma_i}$$

$$= \frac{z - \frac{CPLIM_i}{CPP_i}}{\sigma_i} \tag{5}$$

$Pr_i (xr)$ is calculated for i = 1, 2, .. , 380, covering all channels. The probability that all channel powers are less than their respective limits, that is, no channel exceeds its limit, is given by

$$Pr_0\,(xr) = \prod_{i=1}^{380} Pr_i(xr) \tag{6}$$

Figure 1 illustrates schematically the concept of taking the product of the probabilities to ensure that none of the channels exceeds the limit. The solid curve is the product of all 380 dotted curves. Only five dotted curves for the "worst" channels are shown. Usually only the "worst" ten or so channels with the least margins will numerically contribute to the product. Note that the variable xr (the x-axis) can be interpreted as the fractional reactor power. Our primary interest is at $xr = 1.0$. The reason that a range of xr is covered will be clarified below.

The subscript 0 here denotes "zero" channel exceeds its limit.

The probability that at least one channel exceeds its limit is simply

$$Q_0\,(xr) = 1 - Pr_0\,(xr) \tag{7}$$

The probability density function that some channels exceed their limits is then obtained by taking the derivative of $Q_0\,(xr)$:

$$f\,(xr) = \frac{d}{d(xr)}\, Q_0\,(xr) \tag{8}$$

The next step is to include the common random uncertainty due to the bulk power normalization in the probability consideration. Define σ_{cr} as the one-sigma channel-common uncertainty.

Assuming that the mean common mode bias error has already been included in μ_i , the probability density function for the reactor power error having a value of y (in the same scale as xr) due to the common random uncertainty is given by

$$g\,(y) = \frac{1}{\sqrt{2\pi}\;\sigma_{cr}}\, e^{-\frac{1}{2}\left(\frac{y}{\sigma_{cr}}\right)^2} \tag{9}$$

The probability density function of at least one channel exceeding its limit, accounting for both channel independent and channel common random uncertainties, is then the convolution of f (xr) with g (y), defined as

$$\text{Con f}\,(xr) = \int_{-\infty}^{+\infty} f\,(xr - y) * g\,(y)\;dy \tag{10}$$

Figure 2 illustrates the convolution results. The solid curve denotes f(xr) given by Equation (8), is obtained numerically by taking the derivative of Q_0 (xr). At every point (xr) on the this solid curve, a Gaussian function g(y) is superimposed; that is, we compute the sum of f(a) weighted by g(b) with all possible combinations of a and b such that $a + b = xr$. The convolved function Con f(xr) is shown as the dotted curve. Note that convolution generally tends to broaden the probability density and lower the peak.

The convolved probability that at least one channel exceeds its limit, Con Q_0 (xr), is then obtained by

$$\text{Con } Q_0(xr) = \int_{-\infty}^{xr} \text{Con } f(x)\, dx \tag{11}$$

This is obtained by numerically integrating for the area under the dotted curve in Figure 2 from $-\infty$ to xr.

The convolved performance probability Con P_0 (xr) is simply

$$\text{Con } P_0(xr) = 1 - \text{Con } Q_0(xr) \tag{12}$$

The probability that no channel exceeds its limit at 100% FP is given by Con P_0 (xr=1). This is what we seek as the indicator of channel power performance. Figure 3 shows Con P_0 (xr) (the dotted curve) in contrast to P_0 (xr) (the solid curve) reproduced from Figure 1. **Note that Figures 1 to 3 are schematic and illustrative only and do not represent the actual performance statistics from an operational core state.**

The proposed formulation to calculate a performance probability is now complete. This formalism can easily be extended to compute the probability that one and only one (or two and only two) channel exceeds its limit and that the other 379 channels do not exceed their respective limits.

Ignoring the channel common uncertainty for the time being, the probability that channel i exceeds its limit and the other 379 channels are less than their respective limits is given by

$$Q_{1,i}(xr) = (1 - Pr_i(xr)) * \prod_{j \neq i} Pr_j(xr) \quad .$$

The probability that one and only one channel exceeds its limit, whereas all other channels are less than their respective limit is obtained by summing over i:

$$Q_1(xr) = \sum_i Q_{1,i}(xr) \quad .$$

To account for the channel common uncertainty, the convolution of the derivative of Q_1 (xr) with g (y) is done in a similar fashion described above to obtain Con Q_0 (xr).

3. TREATMENT OF REFUELLING XENON TRANSIENT EFFECTS

The power distribution computed at the surveillance time usually corresponds to a steady state in the sense that xenon transients in refuelled channels have subsided. Immediately after refuelling, the fresh bundles are free of fission products (most notably xenon). Xenon then builds up and settles at its equilibrium level after about 30 hours. The computed powers at steady state do not represent the transient peak powers. Correction factors, defined as the ratio of the peak power after refuelling to the steady-state power, have been derived for the refuelled channel itself and its eight neighbours (see, for example, Reference 2).

The current monitoring practice at PLGS tracks the excursions beyond the administrative action points for transient as well as for steady-state powers. Correction factors are applied to all channels refuelled in the time period from the previous to current surveillance calculation, and similarly for the neighbours of the refuelled channels. The channel power map thus obtained represents the estimated maximum powers between the two surveillance times.

Within the probabilistic framework described in Section 2, two components of xenon corrections are considered. The power peaking derived as the average over the channels in specific core regions is applied as a bias correction factor (μ_{xe}) and is included in the μ_i for the affected channels. The one-sigma variation (σ_{xe}) of the correction factors about the the mean value (μ_{xe}) is treated as a random uncertainty component and is combined in a root-mean-square fashion with the channel random uncertainties, σ_i , for the affected channels. Obviously μ_{xe} and σ_{xe} decay to zero with a characteristic time constant. At a given time instant, t , between surveillance calculations, an instantaneous power map is constructed taking into account the channels refuelled up to time t , and the elapsed time since refuelling to time t for each of the refuelled channels. The instantaneous probability is then computed accordingly. Average probabilities over the time interval between surveillance times can also be computed, as well as time-integrated values.

4. MAIN FEATURES OF METHODOLOGY

The current deterministic approach tallies the number of channels exceeding pre-specified administrative action points. There is a sharp cut-off below the 2-σ administrative level. There is a 1-σ resolution for the channels within the administrative action points. There could be quite a few channels just below the administrative action points and, hence, not counted. Channels within an administrative action point are treated equally, regardless of their inherent margin. In the probabilistic approach presented above, the mathematical formulation is rigourous in the sense that all channels are accounted for with their individual standard normal deviate (see Equation 5), and that there are no hard administrative action points against which to compare. Channels with large deviates will automatically stop contributing to the probability being computed.

The performance indicator here is the probability that no channel exceeds its limit, regardless whether there are any channels above the administrative action points. This is in direct contrast (and a more stringent condition) to using the probability that only the most limiting channel (the channel with the least margin, usually the highest power channel) will not exceed its limit. The latter would be derived solely from the left-most dotted curve in Figure 1. As is evident from Figure 1, the product (solid) curve could be considerably worse than the first dotted curve, depending on the positions of the 2nd, 3rd ... dotted curves. Obviously, when more than one channel is taken into consideration, the performance probability will become lower; and the distribution of the margins to limit for the worst few channels becomes very relevant. Note that it could also happen that the 2nd, 3rd.... channels have very large margins and, therefore, are far away from the worst channel. In this case, then, the solid curve and the first dotted curve would be overlapping. The proposed probabilistic framework covers all these cases as they appear.

Note also that the solid curve in Figure 1 generally does not correspond to a Gaussian distribution and, therefore, cannot be characterized by a sigma value. In the extreme case that the solid curve coincides with the first dotted curve (the other channels do not contribute), the convolution procedure defaults to a direct root-mean-square combination of two Gaussian uncertainties, which results in a sigma given by $\sqrt{\sigma_i^2 + \sigma_{cr}^2}$.

The proposed framework also allows transient xenon effects to be treated more rigorously. The instantaneous power map is based on the time of refuelling and the characteristic build-up of the fission products and, therefore, this approach eliminates the over-conservatism in the current monitoring practice where the peak xenon corrections are applied simultaneously to all affected channels.

5. SAMPLE APPLICATIONS

The *COMPROB module in RFSP calculates channel-power and bundle-power performance probabilities for the power distribution obtained at the time of the surveillance mapping calculation, and for the instantaneous power distributions before and after each channel refuelling that has taken place since the time of the previous surveillance calculation. The method is demonstrated with two sets of site data from PLGS – the first for the month of August 1994 and the second for the first quarter of 1996. It should be noted the magnitudes of the uncertainties used are typical values selected for illustrative purposes only, and they do not represent accepted or recommended values.

For the month of August 1994 (3966 FPD to 3989 FPD), there were eight surveillance calculations. Table 1 shows the tallies of channels within the $0\text{-}\sigma$ to $1\text{-}\sigma$ and $1\text{-}\sigma$ to $2\text{-}\sigma$ range for transient channel powers as would be compiled in the current compliance method. The σ–value used is $\pm 2.7\%$, which represents the total uncertainty (channel random and common random). The bulk-power bias error is +0.25%. Note that there is no credit taken for any temporary power derating that might have taken place. The statistics compiled indicate that there are two refuelling periods (3966 to 3969 FPD and 3978 to 3982 FPD) with a large number of channels having transient powers in the $1\text{-}\sigma$ to $2\text{-}\sigma$ range.

Figure 4a shows the channel power performance probabilities, as computed by *COMPROB. The assumed channel random uncertainty is $\pm 2.0\%$, the common random uncertainty is $\pm 1.0\%$, and the common bias error is +0.25%. The xenon correction factors immediately after refuelling (and assumed to decay with a 9-hour half-time) are given in Table 2. The instantaneous channel power probability (before and after each channel refuelling), and at surveillance times are shown in the figure, together with a horizontal line representing the average over the time interval between surveillance calculations. The performance indicator (the probability that no channel exceeds its limit) is calculated semi-continuously at critical time instants throughout each surveillance period. For the assumed error terms and magnitudes, the interval averaged probability is at 98% or better.

Figure 4b shows the probability over the same time period but with a different channel random uncertainty: $\pm 2.2\%$ instead of $\pm 2.0\%$. All other error terms are kept unchanged. This illustrates the strong sensitivity of the probability to one of the key input variables – the channel random uncertainty term. Figure 5 further illustrates this dependence by focusing on a particular instantaneous core state (at 3980.5 FPD, which has the lowest performance probability of the month) and the average over the surveillance interval from 3978 FPD to 3982 FPD. Note also that power derating has not been credited in these sample calculations.

Figure 6a shows actual PLGS channel-power compliance statistics for the first quarter of 1996. The detailed instantaneous probabilities before and after each refuelling are not shown. The instantaneous probabilities at the surveillance time and the average over the immediately prior interval are shown respectively as solid and empty square data points. The assumed error terms and xenon corrections are summarized in Table 3. They are based on a refuelling transient study using PLGS site data [4]. At 4274 FPD, the error terms were changed to larger values to account for the increase in simulation uncertainty estimated at site for the conditions under which the simulations were done, and this is reflected in the lower probabilities. Also shown in this figure (dotted curve) is the number of transient channel-power excursions beyond the 2-σ administrative action point, with a typical administrative value of $\sigma = \pm 3.23$ %. It is believed that there are excessive conservatisms in the uncertainty allowances shown in Table 3 and in the σ used in the administrative action points.

We should note that the first two months of 1996 were not typical normal operation. In January, PLGS was re-establishing an equilibrium core following a nine-month outage in which there had been significant shutdown fuelling. The high number of excursions in January occurred during the first four days of the month when the unit was returning to high power. The actual channel power limit was never exceeded according to the traditional best-estimate (deterministic) simulation results during this interval.

Figure 6b shows the bundle-power statistics over the same time period. The various error terms for bundle-power performance are also given in Table 3. Generally, performance with bundle power limits is not a concern – the margins to the limits are fairly substantial.

6. SUMMARY

A probabilistic approach to channel-power-limit and bundle-power-limit compliance analysis has been formulated. The mathematical framework is cast in an analogous fashion to the ROPT design- analysis methodology. The performance indicator computed is the probability that no channel (or bundle) power exceeds it limit. In direct contrast to monitoring only the maximum power channel and the probability that the maximum channel power does not exceed the limit, the proposed approach accounts simultaneously for all channels and bundles. Therefore, the proposed method and the performance indicator represent the most stringent conditions to be applied for monitoring purposes.

Xenon transients of the fresh bundles in the refuelled channels are treated more rigorously. The time variations of the xenon build-up and the decay of the temporary power peaking seen immediately after refuelling is accounted for according to the elapsed time from actual refuelling of the channel.

The proposed method has been demonstrated in sample applications using actual PLGS operation data. The various error terms used in these sample applications, however, do not represent the final accepted values. Sensitivity to one of the key input error terms has been illustrated.

All current compliance procedures, including the proposed method, are retroactive. If non-compliance is uncovered at the surveillance time, it could have already corrected itself in the meantime. Therefore, to improve the process, continuous off-line mapping is being contemplated. This could be linked to the data acquisition system to receive up-to-the-minute operating core-parameter data, and thereby, provide a tool in the strict sense of monitoring. A compliance procedure, such as the present one, can then be applied continuously and degrees of non-compliance can be dealt with appropriately, e.g. as operational impairments with appropriate response times.

7. REFERENCES

1. B. Rouben, "Overview of Current RFSP-Code Capabilities for CANDU Core Analysis", Proceedings of ANS Meeting at Philadelphia, 1995 June, TANSAO, p.339.
2. D.A. Jenkins, A.C. Mao, A.S. Gray, "Tracking the Effect of Saturating Fission-Product Build-up in Fresh Fuel on the CANDU Power Distribution", Proceedings of Advances in Nuclear Fuel Management–II, ANS Topical Meeting, to be presented 1997 March.
3. C. Newman, "Channel and Bundle Power Correction Factors For Freshly Fuelled Channels", PLGS Internal Report PIR-95-02, 1995 April.

Table 1 Point Lepreau August 1994 –
Transient Channel Power Compliance Statistics

Surveillance Time (FPD)	3966.2	3969.1	3973.1	3975.3	3978.1	3982.2	3985.1	3989.1
Channels Refuelled Prior to Surveillance Calculation		G13 P16 K04 P11 G09 J15 R17	S03 E16 G05 S08 N13	D14 K06 M22 W09 S15 J09 J19 Q05 B11 L12	M03 T13 D12 M10 R19 F18 N08	N17 F16 N05 U11 F07 P14 F13 J16 H04	U07 M03 O11 T18 B15 E04 N19 G11	R11 R03 P19 K09 D19
Channels with CP within 2.7% of limit		–	–	–	–	–	–	–
Number of channels within 2.7% of limit		0	0	0	0	0	0	0
Channels with CP within 2.7–5.4% of limit		O16 P11 Q10 G08 G09 H15 J15 K15	N13	–	N09 N08 N07	N17 N16 O16 G15 N05 M05 O05 P14 P13 O14 H15 J16 J15	O11 G12 G10	–
Number of channels within 2.7–5.4% of limit		8	1	0	3	13	3	0

Table 2 Xenon Correction Factors Used in Analysis of August 1994 Data

	Refuelled Channel	Near Neighbours	Diagonal Neighbours
Inner Core-Region Channel			
Average (μ_{xe})	+2.16%	+1.27%	+1.12%
1-σ Variation (σ_{xe})	±0.55%	±0.64%	±0.62%
Outer Core-Region Channel			
Average (μ_{xe})	+3.20%	+2.03%	+1.80%
1-σ Variation (σ_{xe})	±1.29%	±1.44%	±1.48%

Table 3 Error Terms and Xenon Correction Factors Used in Analysis of Jan – Mar 1996 Data

	Refuelled Channel	Near Neighbours	Diagonal Neighbours
Inner Core-Region Channel			
Average (μ_{xe})	+4.50%	+2.50%	+2.20%
1-σ Variation (σ_{xe})	±0.21%	±0.21%	±0.24%
Outer Core-Region Channel			
Average (μ_{xe})	+5.00%	+2.90%	+2.50%
1-σ Variation (σ_{xe})	±0.60%	±0.58%	±0.69%
Inner Core-Region Bundle			
Average (μ_{xe})	+5.70%	+3.00%	+2.50%
1-σ Variation (σ_{xe})	±0.20%	±0.20%	±0.20%
Outer Core-Region Bundle			
Average (μ_{xe})	+6.50%	+3.60%	+3.00%
1-σ Variation (σ_{xe})	±0.40%	±0.50%	±0.65%

Typical Error Term Values:

Common Bias Error	+ 0.26%
Channel Random Uncertainty	±2.82%
Bundle Random Uncertainty	±3.39%
Channel/Bundle Common Uncertainty	±1.54%

Figure 1 Probability That Channel Power Is Less Than Its Limit

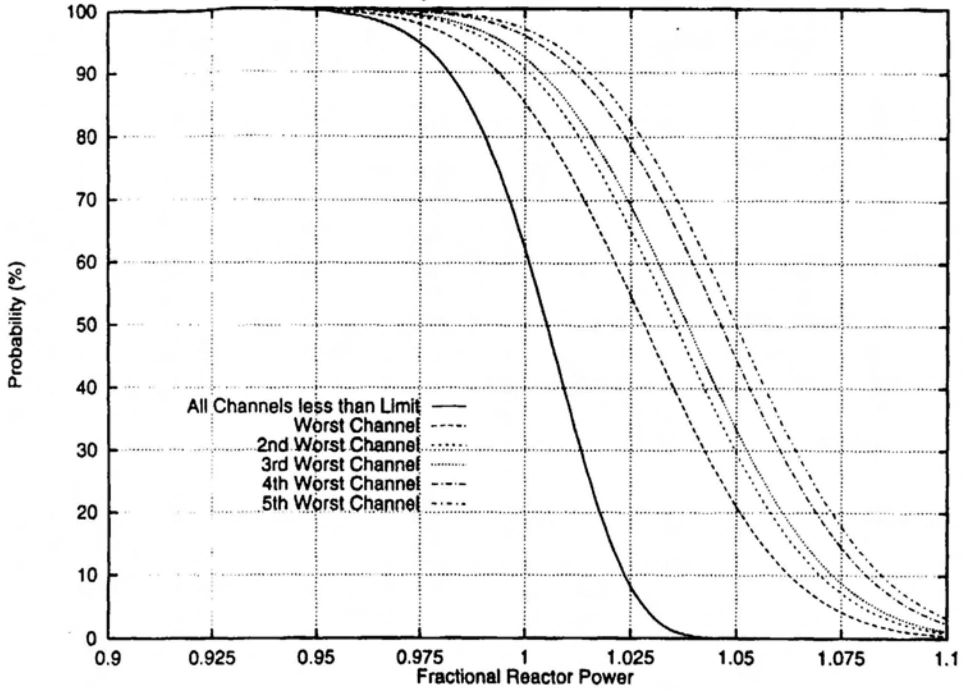


Figure 2 Probability Density That Some Channels Exceed Their Limit

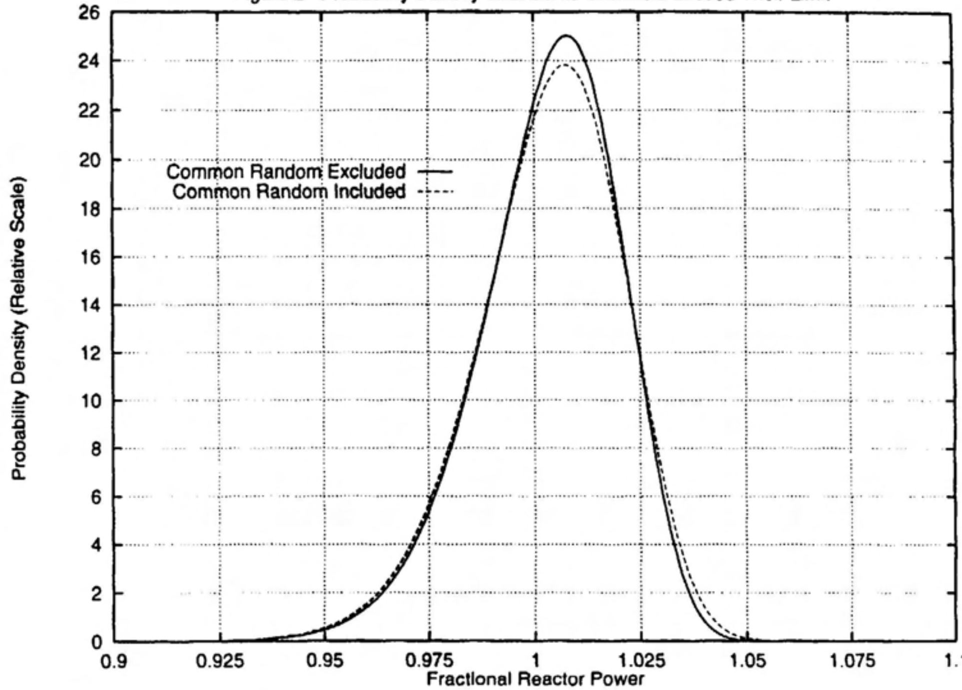


Figure 3 Compliance Probability - That No Channel Exceeds Its Limit

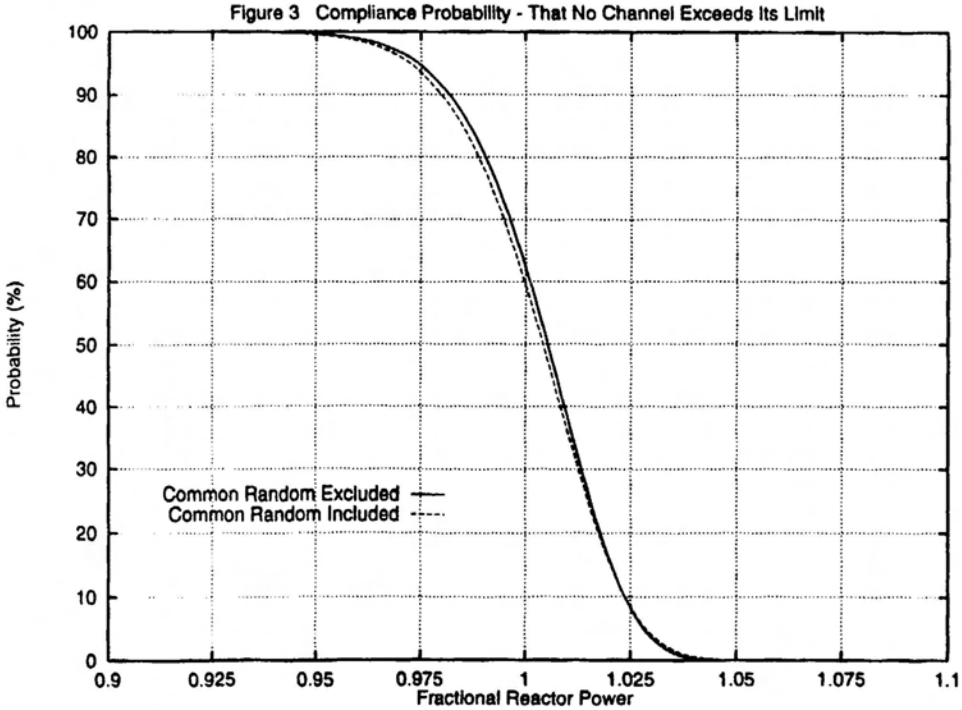
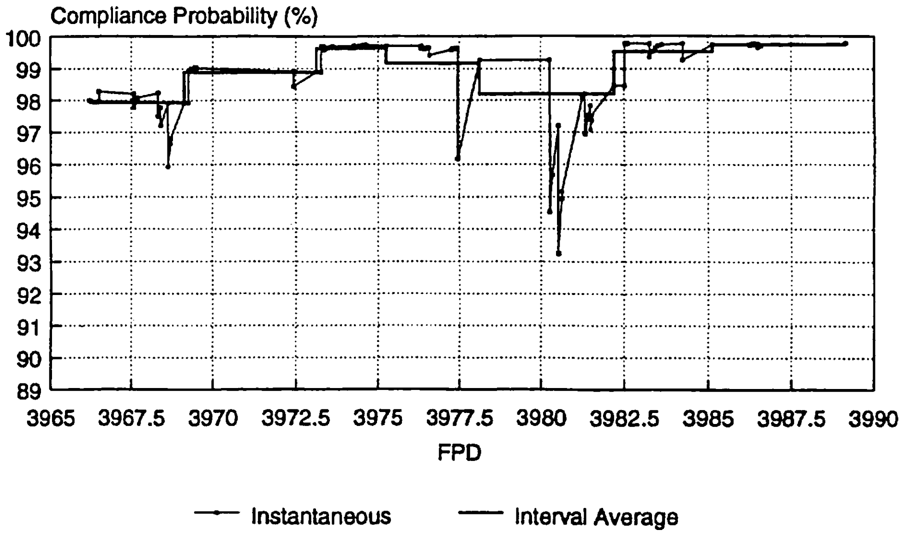
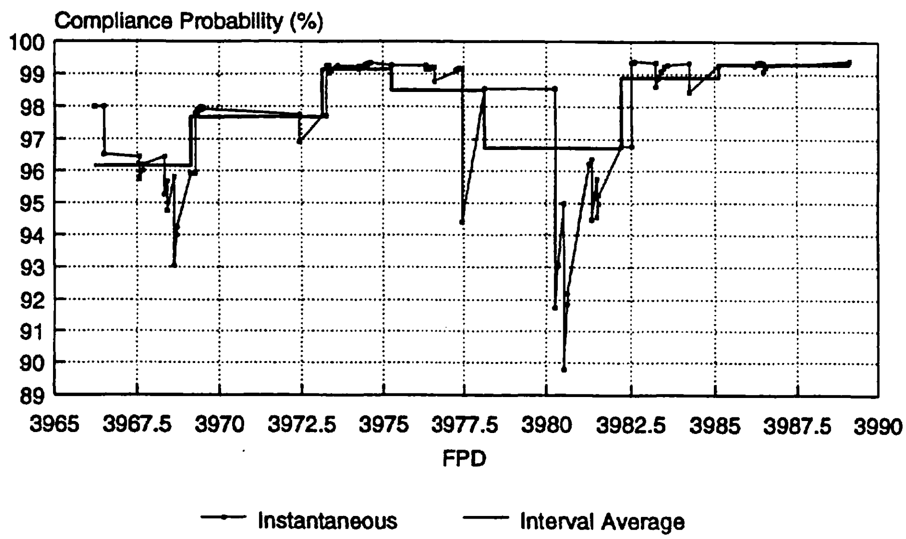


Figure 4a Point Lepreau 94 August
Channel Power Compliance



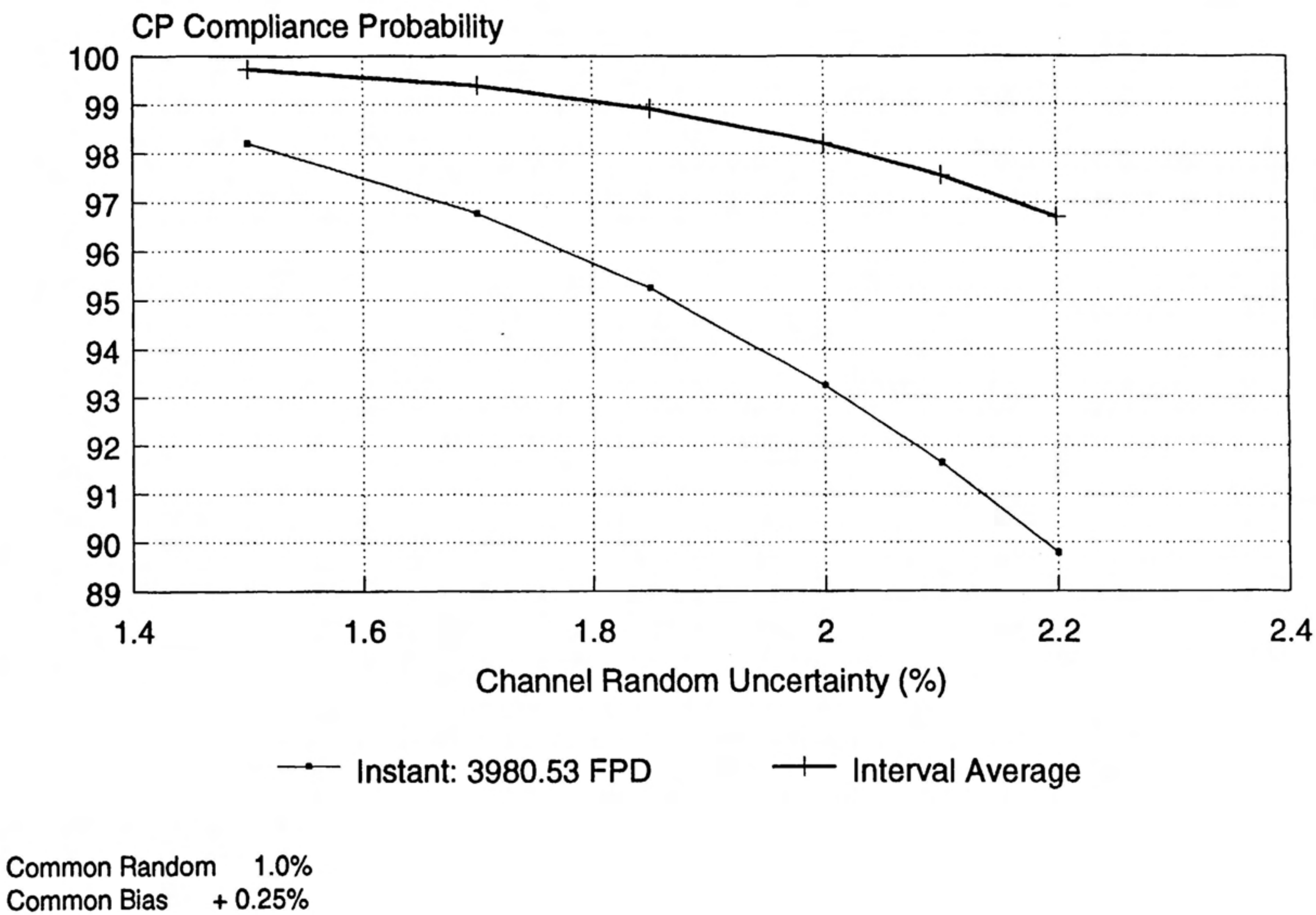
Channel Random 2.0%
Common Random 1.0%
Common Bias + 0.25%

Figure 4b Point Lepreau 94 August
Channel Power Compliance



Channel Random 2.2%
Common Random 1.0%
Common Bias + 0.25%

Figure 5 Channel Power Compliance
Dependence on Channel Random Uncertainty



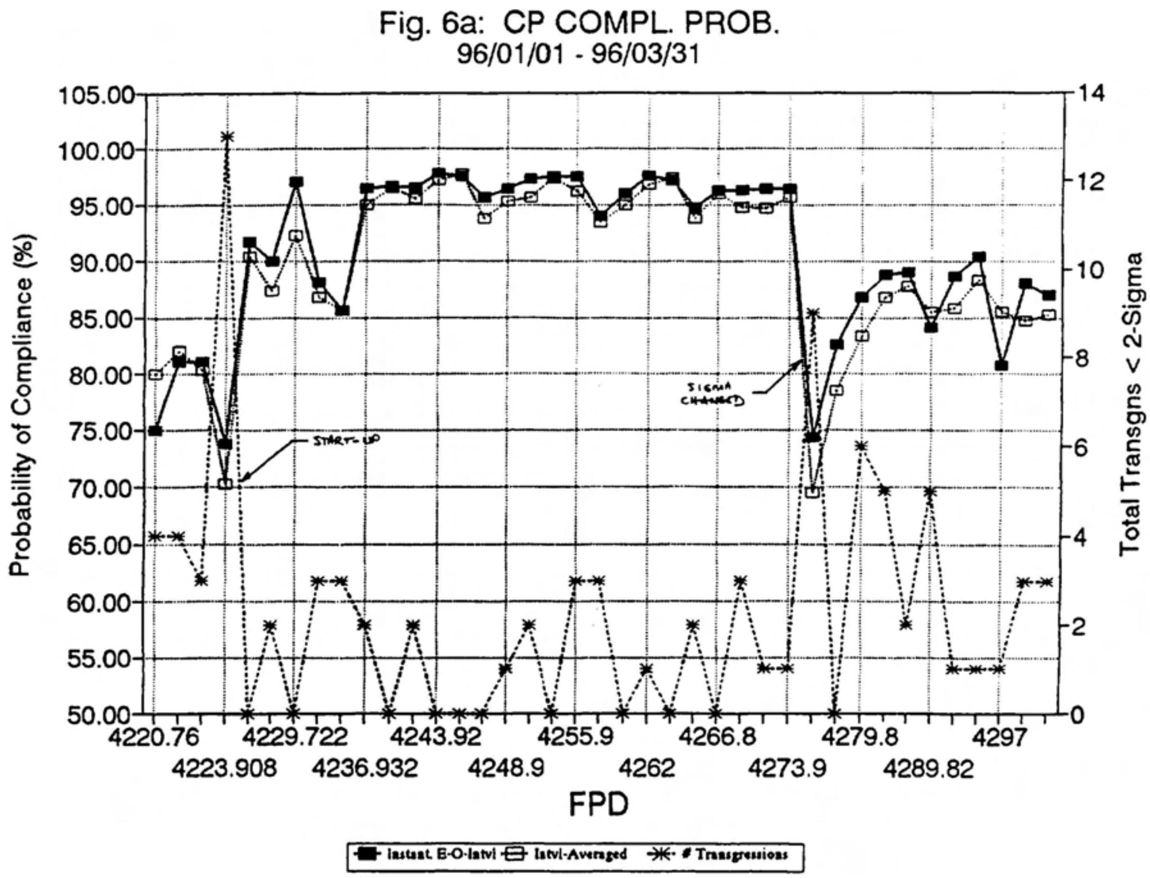


Fig. 6a: CP COMPL. PROB.
96/01/01 - 96/03/31
Probability of Compliance (%)
Total Transgns < 2-Sigma
FPD
START-UP
SIGMA CHANGED
Instant. E-O-Intvl
Intvl-Averaged
Transgressions

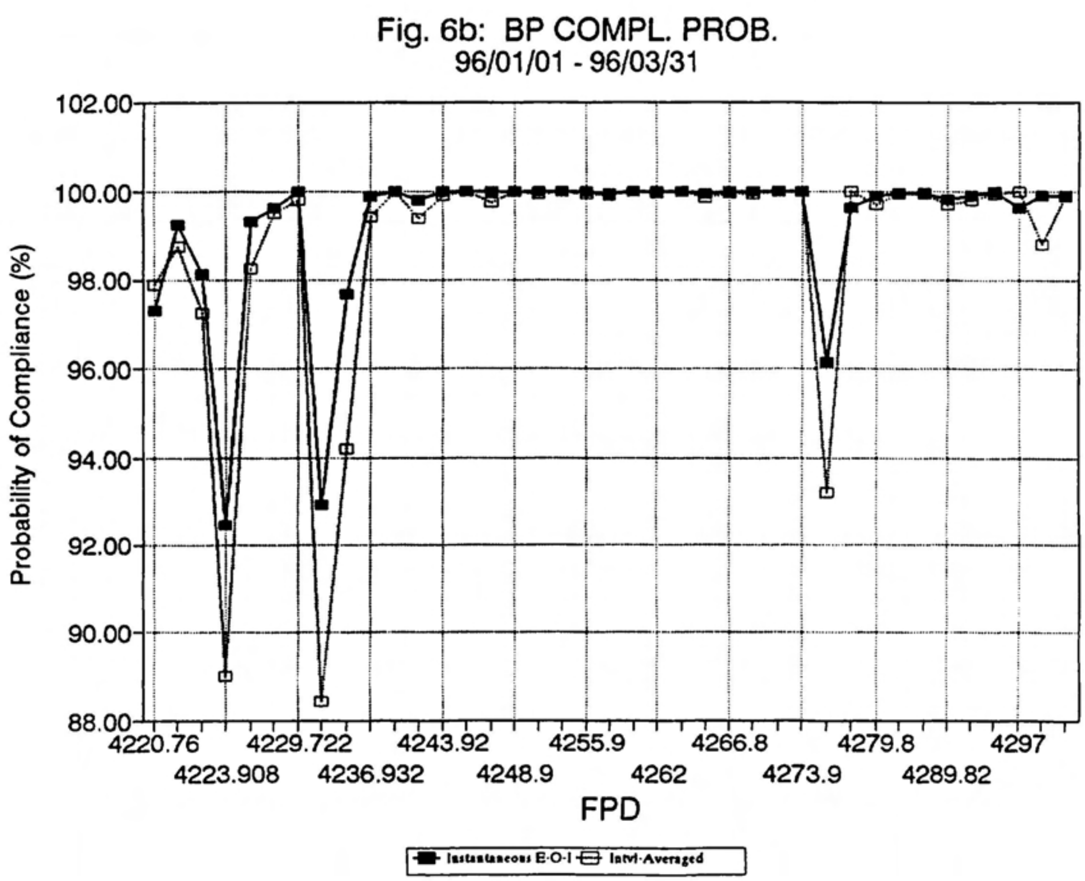


Fig. 6b: BP COMPL. PROB.
96/01/01 - 96/03/31
Probability of Compliance (%)
FPD
Instantaneous E-O-I
Intvl-Averaged